



# Visual Maths

-----Word problems in maths made easy

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## Introduction:

We at **Protean Knowledge Solutions** have great pleasure in placing this booklet “Visual Maths” in your hands/ on your screen. This booklet is about solving word or story problems in maths using simple diagrams. This method is extensively used in Singapore schools up to grade 6.

In the first chapter, basic concepts in mathematics are presented using simple diagrams so the mathematical ideas can be ‘seen’. This visualization is very useful when we are trying to solve word problems. In the later chapters, possible problem scenarios are represented diagrammatically. At the end of each chapter exercise problems are given for practice.

Problem solving is core to learning mathematics; obviously this fact is emphasized in many mathematics curricula around the world.

The process of solving challenging word or story problems helps students to:

- Hone their computational skills
- Reinforce conceptual understanding
- Connect maths with real life situations
- Develop ability to think critically, reason, and communicate
- Develop ability to apply problem solving skills to unfamiliar situations
- Develop curiosity, confidence, perseverance, and open mindedness
- Develop metacognition

Having knowledge of the content and computational ability is one thing and deploying that knowledge to solve word or story problems is a totally different ball game. It needs the ability to - analyze the problem, understand the issues, devise a plan for resolving the problem, execute the plan, and verify that the plan has worked.

Now, there are many strategies to solve a word problem or a story problem. These strategies - among many more - include: guess and check, work backwards, look for a pattern, draw a diagram.

In this booklet, emphasis is on problem solving through diagrammatic representation of the information provided in the problem. This diagrammatic representation helps to translate the word problem into its mathematical representation. If we know how to represent the problem, solving it should be relatively easy.

It is hoped that this booklet will benefit all the educators and the parents who are new to, or only partially exposed to problem solving through diagrammatic representation. The student community will, of course, benefit the most from the lessons incorporating this approach towards the learning process

## Chapter # 1: Introduction to the diagrammatic representation

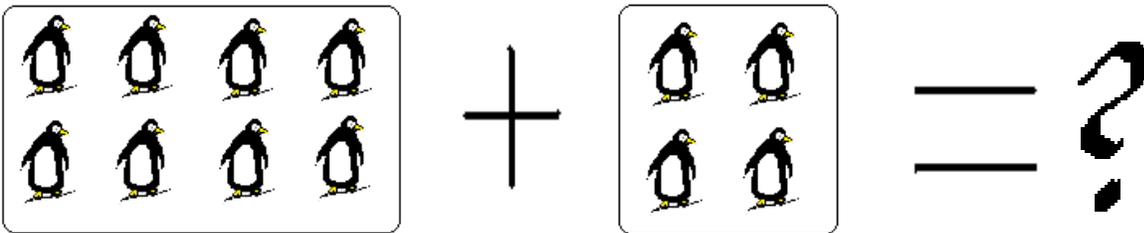
This chapter demonstrates how diagrams can be used to represent basic math concepts and how the diagrams can be made more abstract as we progress through the grades. This chapter also deals with the characteristics of good word problems and how to compare two quantities.

### Addition and Subtraction:

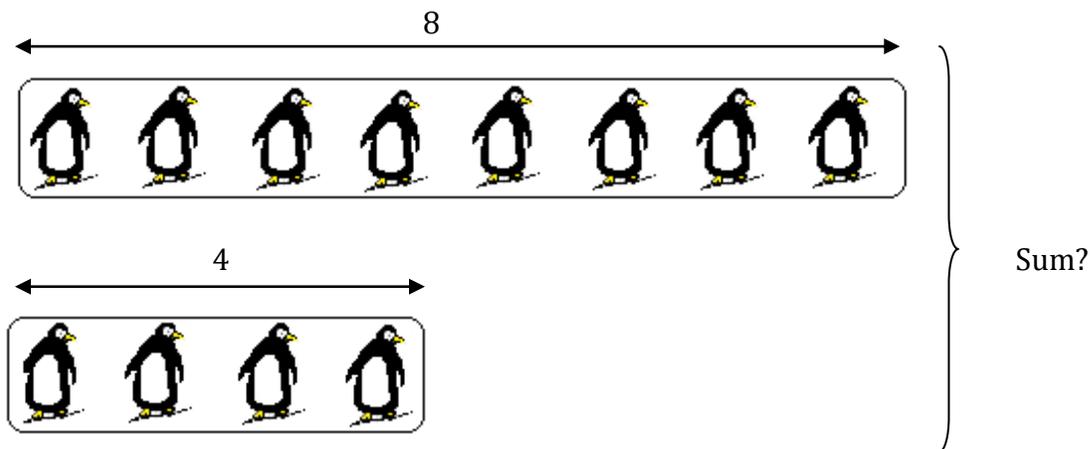
Let us dive into this with a bunch of South Pole residents:

At some place in the South Pole there are 8 penguins swimming. If 4 more penguins join in, how many penguins will be there in all?

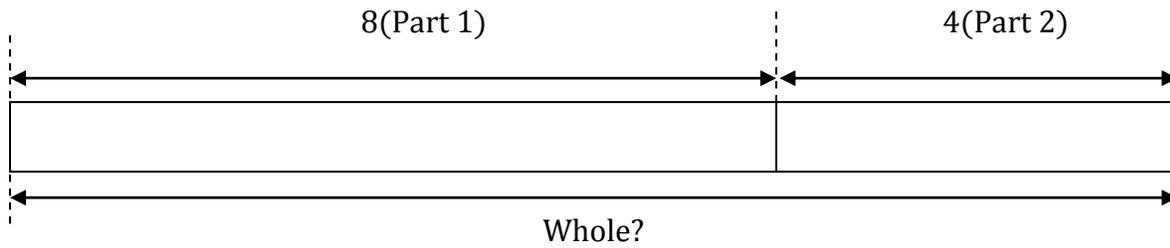
A student in Standard 1 will be given a diagram like the one below:



For a student in Standard 2 the problem will look more like a strip or bar diagram:

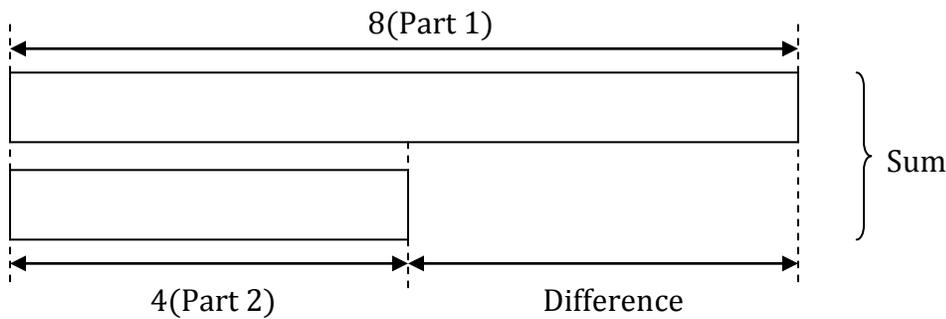


For student in Standard 3, the problem will look something like this:



The diagram above is called "Part-Whole" diagram or "Part-Part-Whole" diagram. In this diagram it is easy to understand the relationship between the "parts" and their corresponding "whole".

The same penguin problem can be represented as:

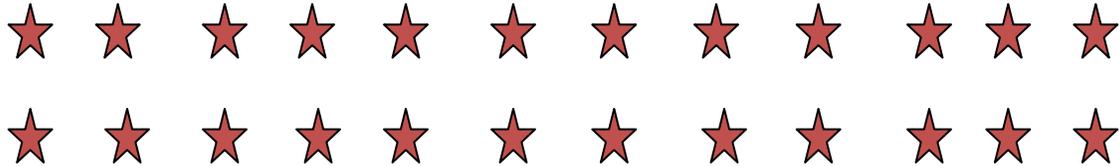


The diagram above is called "Comparison Diagram". Comparison, because the difference in the quantities is easy to see and can be marked on the diagram.

### Multiplication and division:

To introduce the concept of grouping, a student in Standard 1 might be asked to make groups:

For example:



In the diagram above, make groups of three stars. How many groups are there? OR

Share the stars among 2 boys and 2 girls. How many stars will each child get? Circle the stars to show your answer.

For a student in Standard 2 the problem might look something like this:

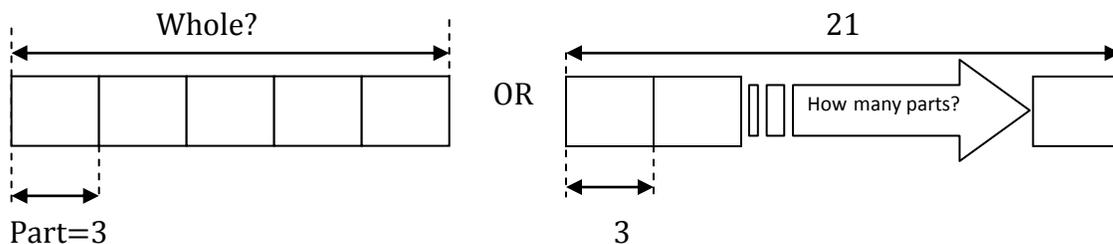
Study the diagram below and write two number sentences about it.



a.)   $\times$   = ? Expected answers are:  $3 \times 5 = 15$  OR  $5 \times 3 = 15$

b.)   $\div$   = ? Expected answers are:  $15 \div 3 = 5$  OR  $15 \div 5 = 3$

For a student in Standard 3 the problem above might look like:



There is one more diagram called “State Transition Diagram”. This type of diagram is suitable when some key element or data in a problem undergoes a change. It will be discussed in the next chapter. The following chapters will take a look at how to apply these

diagrams to various mathematical concepts like addition, subtraction, multiplication, division, fraction, ratios, and percentages.

### **Characteristics of a “Good Word Problem”:**

Word or story problems provide the much needed context for testing computational skills as well as conceptual understanding.

At the elementary level word problems should be:

- Short and to the point, any extraneous or ambiguous information should be avoided
- They should arouse interest but not distract the students.
- They should be based on plausible situations
- There should be one definite answer, though the number of ways students can arrive at that answer be many

Lot of simple problems should be used when a new concept is introduced. When students demonstrate sufficient command over the topic, challenging two or more step problems should be used to test- the newly absorbed skill along with some previously learned skill or skills.

### **Comparison of quantities:**

Before digging deeper, let us understand how to compare two different quantities or values.

When we say that something is more or less, we are comparing two quantities expressed in similar units. However, in this type of comparison, we are not measuring how different one quantity is from another. When we want to measure how different two quantities are, we can do so by calculating:

1. The difference between two quantities(Subtract one quantity from other)
2. Express one quantity as a fraction of other or as a multiple of other
3. Express relation between two quantities as a ratio
4. Express one quantity as a percent of other

Let us take an example to understand the issues.

Keyur has 80 books and his brother Kaustubh has 100 books. Comparison of the number of books between the two boys can be done in the following ways:

Mode of comparison	From Keyur's point of view	From Kaustubh's point of view
As a difference	Keyur got 20 books fewer than Kaustubh  (Keyur's books - Kaustubh's books)	Kaustubh got 20 books more than Keyur  (Kaustubh's books - Keyur's books)
As a fraction	Kaustubh's books as a fraction of Keyur's books are:  Kaustubh's books / Keyur's books  $100 / 80 = 5/4$	Keyur's books as a fraction of Kaustubh's books are:  Keyur's books / Kaustubh's books  $80 / 100 = 4/5$
As a ratio	Keyur's Books : Kaustubh's Books  $80 : 100$ , that is  $4 : 5$ on simplification	Kaustubh's Books : Keyur's Books  $100 : 80$ , that is  $5 : 4$ on simplification
As a percentage	Here we match the percent scale to what Keyur has:  $  \begin{array}{ccccccc}  0\% & & & 100\% & 125\% \\  \leftarrow & \text{-----} & \uparrow & \text{-----} & \rightarrow \\  & \text{Number of books} & 80 & 100 & \\  \end{array}  $ Kaustubh's books are 125% of Keyur's books	Here we match the percent scale to what Kaustubh has:  $  \begin{array}{ccccccc}  0\% & & & 80\% & 100\% \\  \leftarrow & \text{-----} & \uparrow & \text{-----} & \rightarrow \\  & \text{Number of books} & 80 & 100 & \\  \end{array}  $ Keyur's books are 80% of Kaustubh's books

## Chapter # 2: Addition and subtraction

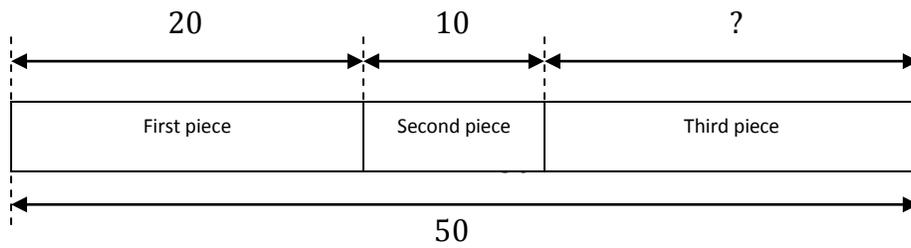
Various possible scenarios in solving problems related to addition and subtraction can be shown as follows:

**Scenario: Given the whole and a part or parts we need to find the remaining part or parts.**

Example:

Suhas cut a 50 cm long wire into 3 pieces. If the first piece was 20 cm long and the other piece was 10 cm long, then how long was the third piece?

Our Part-Part-Whole Diagram will look like this:



The diagram above represents the wire as a whole and its constituent parts. It can be seen easily that the length of the 3<sup>rd</sup> piece can be obtained by subtracting lengths of other two pieces from the whole.

$$20 + 10 + \text{Length of the 3}^{\text{rd}} \text{ piece} = 50$$

$$\begin{aligned}\therefore \text{Length of 3}^{\text{rd}} \text{ piece} &= 50 - (20 + 10) \\ &= 20 \text{ cm}\end{aligned}$$

*Algebraically:*

Suppose  $x$  cm is the length of the third piece, then we can write following equation:

$$x + 10 + 20 = 50$$

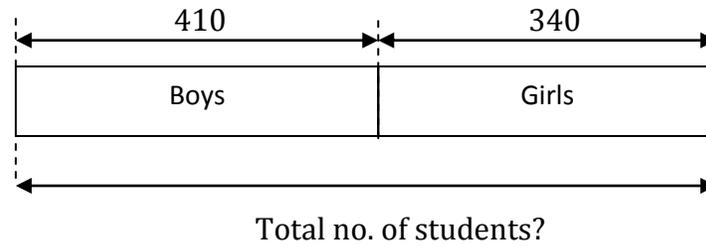
$$\begin{aligned}\therefore x &= 50 - (20 + 10) \\ &= 20 \text{ cm}\end{aligned}$$

**Scenario: Given parts find the whole.**

Example:

There are 340 girls and 410 boys in the school. So, how many students are there altogether?

This type of problem can be easily visualized with the Part-Part-Whole Diagram. The diagram will look something like given below:



Total number of students = number of girls in the school + number of boys in the school.

$$\therefore \text{No. of students} = 410 + 340 = 750$$

*Algebraically:*

Suppose  $x$  is the total number of students, then we can write following equation:

$$x = 410 + 340$$

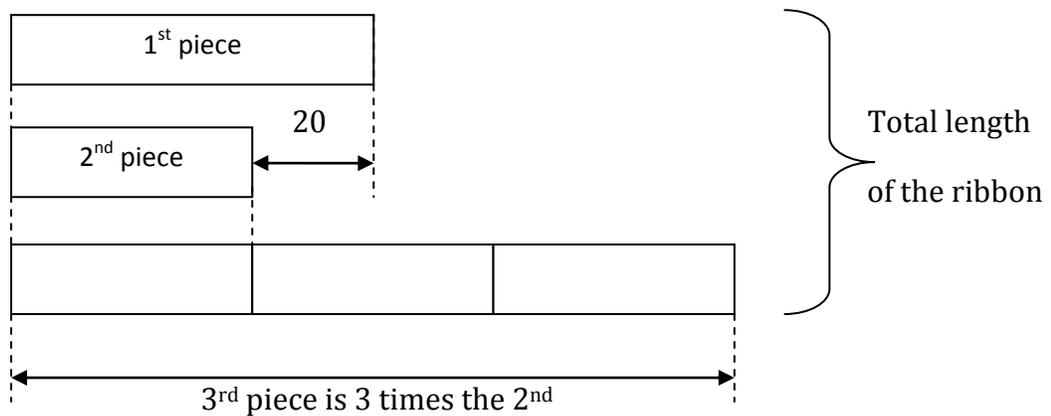
$$\therefore x = 750$$

**Scenario: Given relationship between the parts find the intermediate unknowns and then the whole.**

Example:

A ribbon was cut into 3 pieces. The first piece of ribbon was 20 cm longer than second piece. The third piece was three times as long as the second piece. If the first piece was 60 cm long then what was the length of the third piece? What was the length of the ribbon before it was cut?

Let us use the “Comparison diagram” for this problem and represent the information provided as below:



$$\text{Length of 2}^{\text{nd}} \text{ piece} = \text{Length of 1}^{\text{st}} \text{ piece} - 20$$

$$\therefore \text{Length of 2}^{\text{nd}} \text{ piece} = 60 - 20 = 40 \text{ cm}$$

$$\therefore \text{Length of 3}^{\text{rd}} \text{ piece} = 40 \times 3 = 120 \text{ cm}$$

$$\therefore \text{Length of the ribbon was: } 60 + 40 + 120 = 220 \text{ cm}$$

*Algebraically:*

Suppose  $x$  is the length of the 2<sup>nd</sup> piece then we can write following equations:

$$\text{Length of the 1}^{\text{st}} \text{ piece} = \text{Length of the 2}^{\text{nd}} \text{ piece} + 20 = x + 20 = 60$$

$$\therefore x = 60 - 20 = 40$$

$$\therefore \text{Length of the 3}^{\text{rd}} \text{ piece} = 3 \times \text{Length of the 2}^{\text{nd}} \text{ piece} = 3x = 3 \times 40 = 120$$

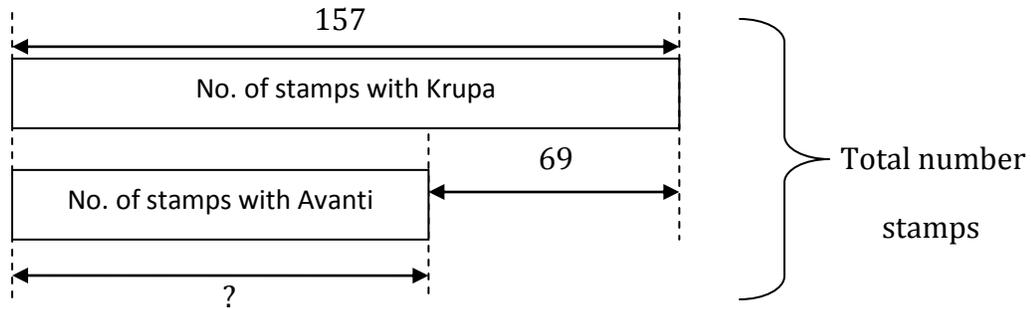
$$\therefore \text{Total length of the ribbon} = 60 + 40 + 120 = 220 \text{ cm}$$

**Scenario: Given a part and a difference, find the other part and the whole.**

Example:

Krupa has 157 stamps. She has 69 stamps more than Avanti, so how many stamps does Avanti have? How many stamps they have all together?

The “Comparison Diagram” will look like the one below:



We can see that:

$$\text{No. of stamps with Avanti} = \text{No. of stamps with Krupa} - 69$$

$$\therefore \text{Stamps with Avanti} = 157 - 69 = 88$$

Therefore, the number of stamps that they have all together is:

$$88 + 157 = 245$$

*Algebraically:*

Suppose  $x$  is the number of stamps with Avanti, we can write the following equation:

$$157 = x + 69$$

$$\therefore x = 157 - 69 = 88$$

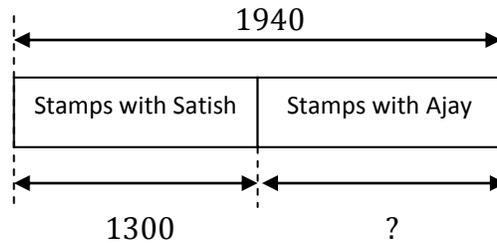
$\therefore$  Number of stamps they have all together is:

$$88 + 157 = 245$$

**Scenario: Given the whole and a part, find the other part and then the difference between the parts.**

Example:

Satish and Ajay have 1940 stamps altogether. If Satish has 1300 stamps then how many more stamps does he have than Ajay?



Find the number of stamps with Ajay first.

$$\begin{aligned}\text{Stamps with Ajay} &= 1940 - 1300 = 640 \\ \therefore \text{The difference} &= 1300 - 640 = 660 \\ \therefore \text{Satish has } &660 \text{ stamps more than Ajay.}\end{aligned}$$

*Algebraically:*

Suppose  $x$  is the number of stamps with Ajay. Based on the information given we can write following equation:

$$\begin{aligned}x + 1300 &= 1940 \\ \therefore x &= 1940 - 1300 = 640\end{aligned}$$

To find how many stamps Satish has more than Ajay, we subtract:

$$\begin{aligned}\text{The difference} &= 1300 - 640 = 660 \\ \therefore \text{Satish has } &660 \text{ stamps more than Ajay.}\end{aligned}$$

Another way of doing this is:

Suppose Satish has  $x$  stamps more than Ajay. Then we can write the following equation:

$$x = 1300 - \text{Stamps with Ajay}$$

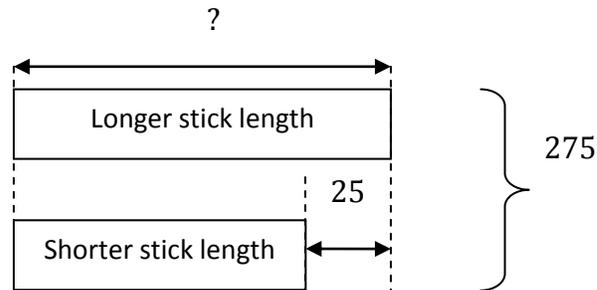
Now,

$$\begin{aligned}\text{Stamps with Satish} + \text{Stamps with Ajay} &= \text{Total} \\ \therefore 1300 + (1300 - x) &= 1940 \\ \therefore 2600 - x &= 1940 \\ \therefore x &= 2600 - 1940 = 660 \\ \therefore \text{Satish has } &660 \text{ stamps more than Ajay.}\end{aligned}$$

**Scenario: Given the whole and the difference between its parts, find parts.**

Example:

The total length of two sticks is 275 cm. If the difference in their length is 25 cm find the length of the longer stick.



Length of longer stick + Length of shorter stick = 275 cm

Since the difference between the length of sticks is 25 cm

$$\begin{aligned}\therefore \text{Length of shorter stick} &= (275 - 25) \div 2 \\ &= 250 \div 2 \\ &= 125\end{aligned}$$

$$\begin{aligned}\therefore \text{Length of longer stick} &= 125 + 25 \\ &= 150 \text{ cm}\end{aligned}$$

*Algebraically:*

Suppose length of the shorter stick is  $x$ .

Then, length of the longer stick =  $x + 25$

Now, the total length of the two sticks is 275

$$\begin{aligned}\therefore (x + 25) + x &= 275 \\ 2x + 25 &= 275 \\ \therefore 2x &= 275 - 25 = 250 \\ \therefore x &= 250 \div 2 = 125 \text{ cm}\end{aligned}$$

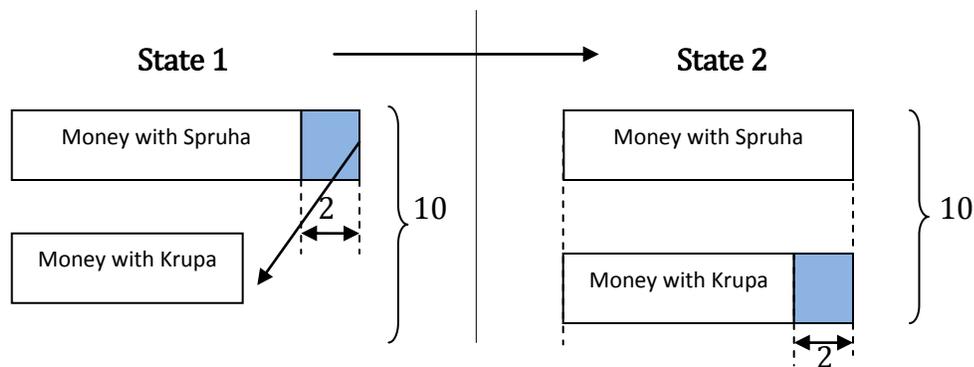
So, the length of the longer stick is:  $x + 25 = 125 + 25 = 150 \text{ cm}$

## The State Transition Diagram

The diagrams that have been encountered so far depict one state of the problem. When a change is introduced in the problem, some elements in the problem acquire new values thereby changing the state of the problem. “State Transition Diagram” reflects this transition diagrammatically.

Consider the following example:

Spruha and her sister Krupa had Rs. 10/- altogether. After giving her sister Rs. 2/-, Spruha now has same amount of money as her sister. How much money did Spruha have in the beginning?



From the State 2 diagram we can see that:

$$\text{Money with Spruha} = \text{Money with Krupa} = 10 \div 2 = 5$$

$$\therefore \text{Money had by Spruha in the beginning} = 5 + 2 = 7$$

*Algebraically:*

Suppose  $x$  is the amount of money Spruha has in the beginning and  $y$  is the amount of money Krupa has in the beginning.

Now we know that:

$$x + y = 10 \quad \text{-----Equation 1}$$

After giving Rs. 2/- to Krupa

Money with Spruha =  $x - 2$  and it equals new money with Krupa

$$\therefore x - 2 = y + 2$$

$$\therefore x = y + 4 \quad \text{-----Equation 2}$$

Substituting value of  $x$  from Equation 2 in Equation 1 we get:

$$\begin{aligned}(y + 4) + y &= 10 \\ \therefore 2y + 4 &= 10 \\ \therefore 2y &= 10 - 4\end{aligned}$$

Solving for  $y$  we get:

$$\begin{aligned}y &= 6 \div 2 = 3 \\ \therefore x &= y + 4 = 7\end{aligned}$$

Spruha had Rs. 7/- in the beginning.

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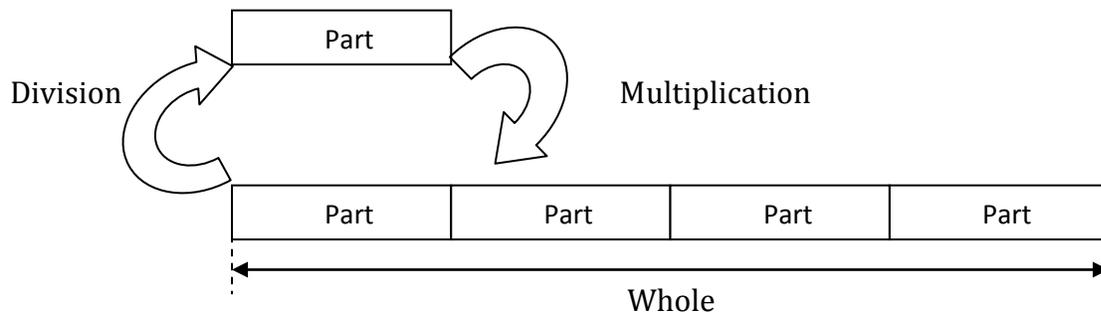
**Exercise:**

1. The price of a papaya and a watermelon is Rs. 80/- altogether. If the watermelon costs Rs. 20/- more than the papaya, find the price of the papaya.
2. In our neighborhood a postman delivered 175 letters on Monday. On Tuesday he delivered 40 fewer letters than on Monday. How many letters did the postman deliver on the two days?
3. There are 38 students in a robotics class. There are 6 more girl students than the boys. How many boys are there in the class?
4. A fruit seller has 280 fruits. 120 were mangoes and the rest were apples. After selling some apples and 20 mangoes, he realized that he has same number of apples and mangoes with him. How many apples did he sell?
5. Nikhil bought a book and a compass box. He gave Rs. 200/- to the cashier and the cashier returned Rs. 20/-. If the price of the compass was Rs. 75/- then what was the price of the book?
6. Deepak has 85 marbles. He has 7 marbles more than Sanju and 22 marbles less than Sudheer. How many marbles do they have altogether?
7. A fruit seller had 320 apples of which 128 were red and remaining were green. How many more green apples were there than the red?
8. As a part of festivity 3000 balloons were released. Of these 2148 were orange balloons and the rest were white and green and yellow. If there were equal number of white and yellow balloons, how many green balloons were there?
9. Nina and her sister Tina had, at the beginning, Rs. 100/- altogether. When Nina gave Tina Rs. 20/- both of them had the same amount of money. How much money did Tina have at the beginning?
10. A rope 2850 inches long was cut in three pieces. Of the three pieces, two pieces were equal and the third piece was 450 inches longer than the other pieces. What is the length of the longer piece?

## Chapter # 3: Multiplication and Division

The “Part-Part-Whole Diagram” and the “Comparison Diagram” can be easily extended to the concepts of multiplication and division.

Visualize multiplication as a Part added repeatedly to make a Whole and division as a Whole divided into equal Parts.



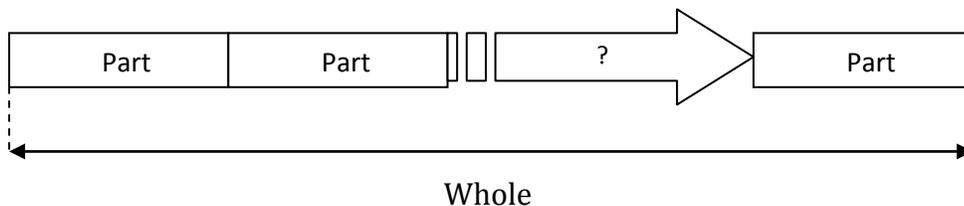
The question: How many “Parts” make the “Whole”? Or The “Whole” is divided into how many “Parts” establishes the quantitative relationship between the “Whole” and the “Part”.

We can express the relationship as follows:

One Part  $\times$  Number of Parts = Whole = Part + Part + ..... + Part

The Whole  $\div$  Number of Parts = One Part

The diagrams shown above works fine when we are given “Number of Parts” which make up the “Whole”. However, when the number is unknown, the diagram has to reflect that. We do this by breaking the “Whole” as shown below:

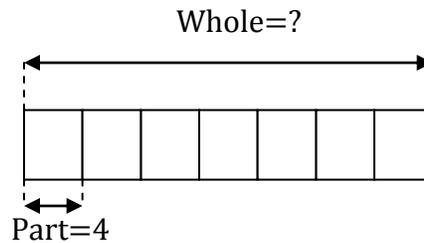


**Scenario: Given one part and number of parts find the whole.**

Example:

On his birthday Anay gave four candies to each of his seven friends. So, how many candies did Anay have in the beginning?

Here diagram of the problem looks something like shown below:



$$\begin{aligned}\text{Candies Anay had in the beginning} &= \text{Number of Parts} \times \text{One Part} \\ &= 7 \times 4 = 28\end{aligned}$$

*Algebraically:*

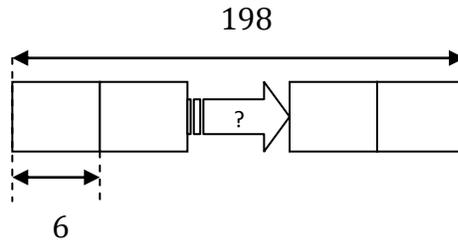
Suppose Anay had  $x$  number of candies in the beginning, from the information given we can write:

$$\begin{aligned}x \div 7 &= 4 \\ \therefore x &= 4 \times 7 \\ \therefore x &= 28\end{aligned}$$

**Scenario: Given the whole and one part find the number of parts.**

Example:

A baker made 198 biscuits. If he packs 6 biscuits in one packet, how many packets will he make?



$$\therefore \text{Number of packets} = 198 \div 6 = 33$$

*Algebraically:*

Suppose the baker makes  $x$  number of packets, each with 6 biscuits in it,

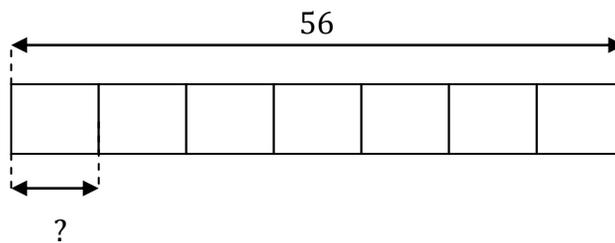
$$\therefore x \times 6 = 198$$

$$\therefore x = 198 \div 6 = 33$$

**Scenario: Given the whole and number of parts find one part.**

Example:

Pramod has 56 marbles which are 7 times as many marbles as his friend Vinod has. Then how many marbles does Vinod have?



$$\therefore \text{Marbles with Vinod} = 56 \div 7 = 8$$

*Algebraically:*

Suppose Vinod has  $x$  number of marbles, then we can write the following equation:

$$x \times 7 = 56$$

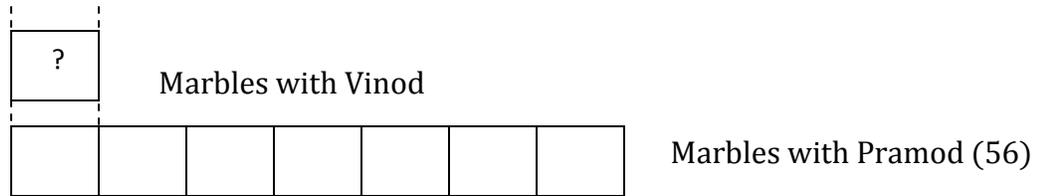
$$\therefore x = 56 \div 7 = 8, \text{ so Vinod has 8 marbles.}$$

**Alternate approach:**

Example:

Pramod has 56 marbles which are 7 times as many marbles as his friend Vinod has. Then how many marbles does Vinod have?

Using Comparison Diagram we can depict the problem as shown below:

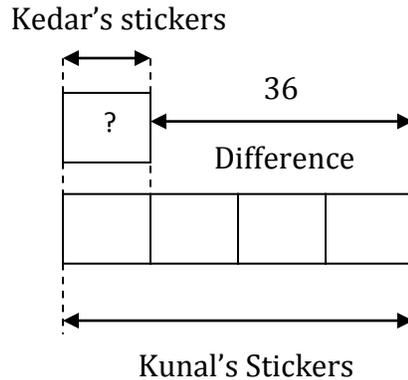


$$\text{Marbles with Vinod} = 56 \div 7 = 8$$

**Scenario: Given the multiple and the difference, find the quantities.**

Example:

Kunal has 4 times as many stickers as Kedar. If Kedar has 36 less stickers than Kunal, then how many stickers does Kedar have?



The difference = One part  $\times$  3

$$\therefore \text{One part} = 36 \div 3 = 12$$

$$\therefore \text{Number of stickers Kedar had initially} = 12 \times 4 = 48$$

*Algebraic ally:*

Suppose  $x$  is the number of stickers Kedar has and  $y$  is the number of stickers Kunal has then from the information we can write,

$$x \times 4 = y \text{ -----Equation 1}$$

And

$$y - x = 36 \text{ -----Equation 2}$$

Substituting for value of  $y$  in equation 2 from equation 1, we get:

$$4x - x = 36$$

$$\therefore 3x = 36$$

$$\therefore x = 36 \div 3$$

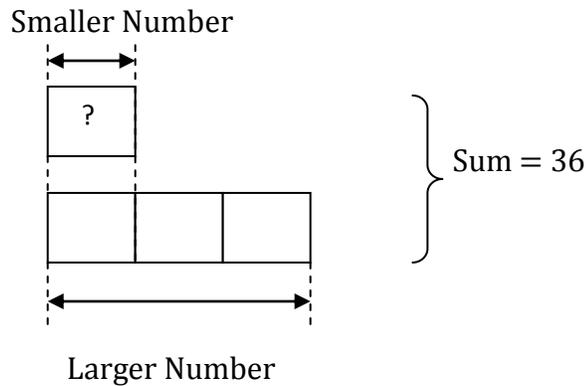
$$\therefore x = 12$$

**Scenario: Given the sum and the multiple find the quantities.**

Example:

The sum of two numbers is 36 and one number is three times the other number. Find the product of the numbers.

We can describe the problem diagrammatically as shown below:



We can see that, 4 parts = 36

$$\text{The smaller number} = 36 \div 4 = 9$$

$$\text{The larger number} = 9 \times 3 = 27$$

$$\therefore \text{The product of the numbers} = 9 \times 27 = 243$$

*Algebraically:*

Suppose  $x$  is the smaller number, then:

$$\text{The larger number} = x \times 3 = 3x$$

Since sum of the two numbers is 36, we can write:

$$x + 3x = 36$$

$$\therefore 4x = 36$$

$$\therefore x = 36 \div 4 = 9$$

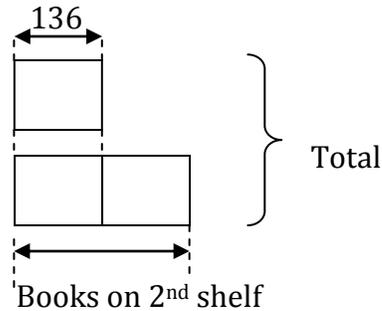
$$\therefore \text{The product of the numbers} = x \times 3x$$

$$= 9 \times 27$$

$$= 243$$

**Scenario: Given a quantity and multiple find other quantity and sum.**

Ulhas has 136 books on the first shelf of his library rack and twice as many books on the second shelf. Find the number of books on the other shelf. Find the total number of books on the two shelves.



$$\text{Books on the second shelf} = 2 \times 136 = 272$$

$$\text{Total number of books} = 272 + 136 = 408$$

OR

$$\text{Total number of books} = 3 \times 136 = 408$$

*Algebraically:*

Suppose  $x$  is the number of books on the second shelf. Then from the information given in the problem we can write:

$$x = 2 \times 136 = 272$$

The number of books on the second shelf equals to 272

$$\therefore \text{The total number of books} = 136 + 272 = 408$$

Another way of looking at the problem is:

Suppose  $x$  is the number of books on the first shelf, then:

$$\text{The number of books on the 2<sup>nd</sup> shelf} = 2 \times x = 2x$$

$$\therefore \text{The total number of books} = x + 2x = 3x$$

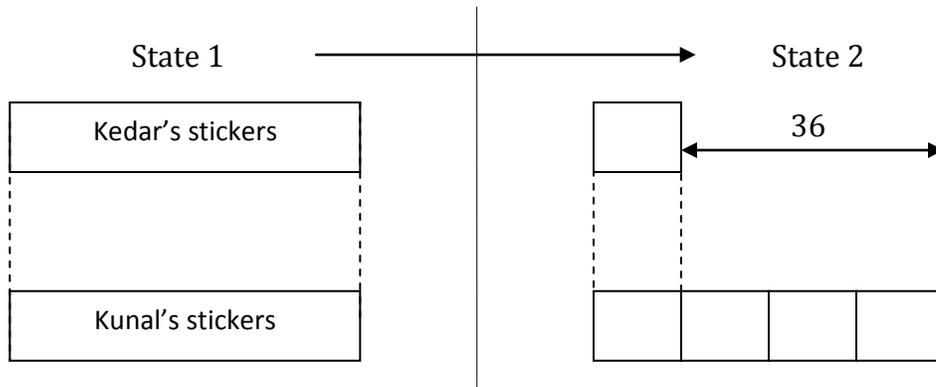
But since  $x = 136$ ,

$$\text{Total number of books} = 3 \times 136 = 408 \text{ and books on 2<sup>nd</sup> shelf} = 2x = 2 \times 136 = 272$$

Example:

Kedar and Kunal, initially, had an equal number of stickers. After Kedar lost 36 of his stickers, Kunal has 4 times as many stickers as Kedar. How many stickers did Kedar have initially?

The change in the state of the problem can be depicted using “State Transition Diagram”.



From the diagram it can be seen that:

$$\begin{aligned} 3 \text{ parts} &= 36 \text{ stickers} \\ \therefore 1 \text{ part} &= 36 \div 3 = 12 \text{ stickers} \\ \therefore \text{Kedar initially had } &12 \times 4 = 48 \text{ stickers} \end{aligned}$$

*Algebraically:*

Suppose  $x$  is the number of stickers that Kedar had initially and  $y$  is what Kunal had. From the information given we can write:

$$x = y \text{ -----Equation 1}$$

After Kedar lost 36 of his stickers, from the information given, we can write:

$$4 \times (x - 36) = y \text{ -----Equation 2}$$

Solving Equation 2 we get:

$$\therefore 4x - 144 = y$$

Substituting for  $y$  from Equation 1, we get:

$$\begin{aligned} 4x - 144 &= x \\ \therefore 4x - x &= 144 \\ \therefore 3x &= 144 \\ \therefore x &= 48 \end{aligned}$$

**Exercise:**

1. A plastic pipe 3 meters 36 cm long is cut in pieces measuring 6 cm each. How many pieces will be there?
2. On a scale Purva weighs 26 Kg and her brother weighs half as much as Purva. If their father weighs twice as much as Purva and her brother put together. Find how much the whole family will weigh on that scale.
3. A farmer packed 378 Kg tomato in crates. Each crate contained 6 Kg tomato. How many crates are there in all?
4. Krishna and his friend Sundar have 60 marbles altogether. If Sundar has three times as many marbles as Krishna, how many marbles does Krishna have?
5. A book costs five times as much as a compass box. If the difference in their price is Rs. 320/- Find the price of the book.
6. Sowjanya and Sunetra went for shopping and, in the beginning, had Rs. 800/- all together. After each bought a book priced at Rs. 75/-, Sowjanya had four times as much money as Sunetra. How much money did Sunetra have in the beginning?
7. Neha was reading a book. On the first day she read 27 pages. Next day onwards she read twice as many pages as what she read on the previous day. If she finished the book on the third day then how many pages did the book have?
8. A farmer packed 1446 pomegranates in boxes for shipping to the market. If there were 6 pomegranates per box, how many boxes were shipped?
9. Uday has 120 stamps. His sister Jyoti has five times as many stamps as Uday. How many stamps did they have altogether?
10. Some girls shared the cost of a present equally. If the present cost Rs. 225/- and each girl contributed Rs. 15/-, how many girls shared the cost?

## Chapter # 4: Fractions

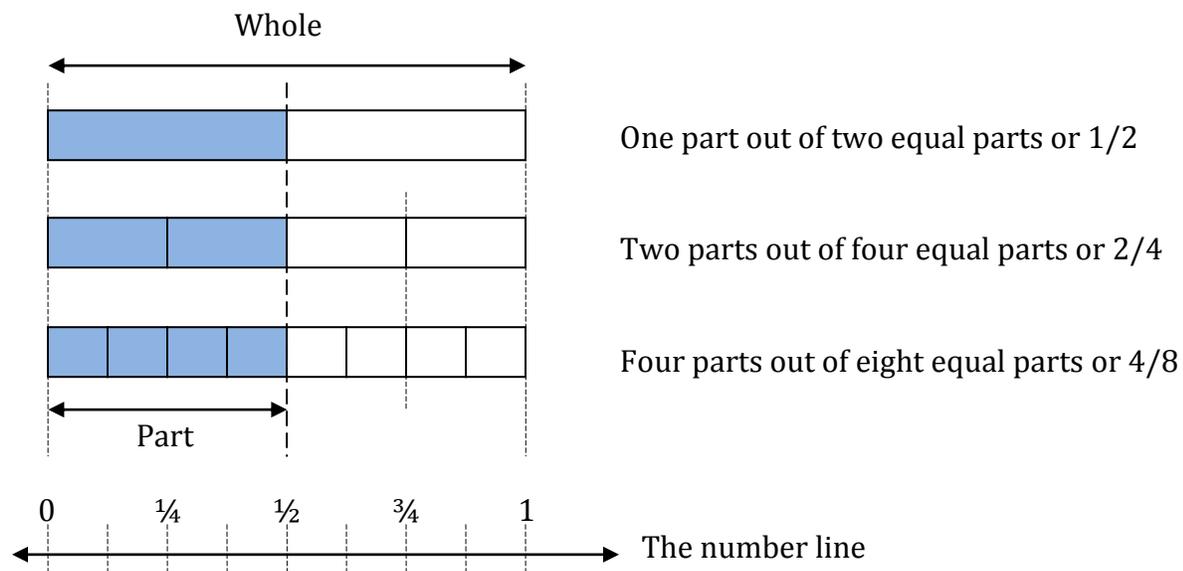
Fraction is a number. It has a place of its own on the number line and like any number, fractions can be added, subtracted, multiplied, and divided. Earlier, like normal multiplication tables, we used to have multiplication tables for common fractions like  $1/4$ ,  $1/2$ ,  $3/4$  etc. These tables can be considered as an excellent vehicle for the reinforcement of our understanding of fractions as numbers. The most commonly encountered description of fractions:

Fraction can be described as a part of a whole where the whole can be one item or a group of items or a region, and the fraction establishes a relationship between the part and its whole.

In fractions, the most important idea that we need to understand is “Equivalent Fractions”. Equivalent fractions represent same part of the whole and occupy same place on the number line, consider example given below:

$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$  These are equivalent fractions. If you refer the diagram below you can see that equivalent fractions represent the same part of the whole.

Diagrammatically we can represent equivalent fractions as:



Both diagrams can be used to understand fraction related problems.

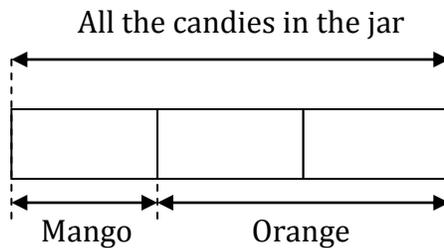
### Part-Part-Whole Diagram

Here parts are expressed as fractions of the whole.

Consider following example:

There are 30 candies in the jar. One third of the candies are Mango candies and the remaining are Orange candies.

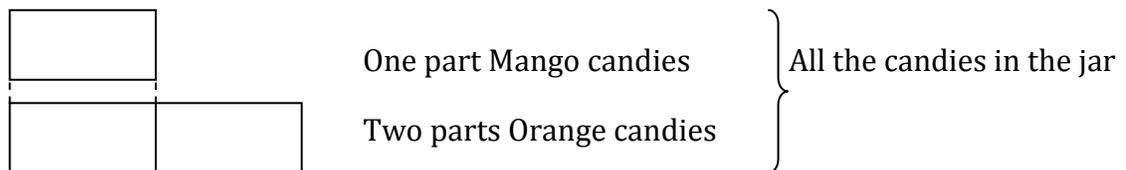
This example talks of two parts. One part is  $\frac{1}{3}$  of the whole and other part is  $\frac{2}{3}$  of the whole. Diagrammatically we can express this as follows



Here, we divided the candies in 3 equal parts. One part represents Mango candies and two parts represent Orange candies.

### Comparison Diagram

The relationship between Mango candies, Orange candies and their sum (“The Whole”) can be visualized in “Comparison Diagram” also.



Here we can express the relationship between the parts as:

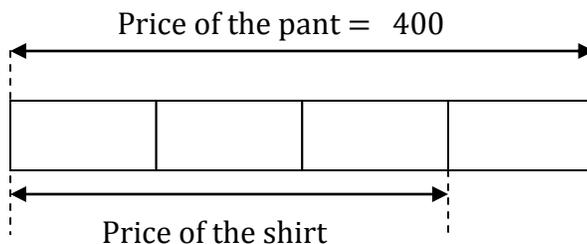
Orange candies are two times Mango candies OR Orange candies are twice as many as Mango candies OR Orange candies are  $\frac{2}{3}$  of the whole.

Mango candies are half of Orange candies OR Mango candies are  $\frac{1}{3}$  of the whole.

**Scenario: Given one quantity and the fraction, find the other quantity.**

Example:

Price of the shirt is  $\frac{3}{4}$  the price of pant. If the pant costs Rs. 400/- then what is the price of the shirt?



Here “price of the pant” is divided into 4 equal parts and we are taking 3 parts to determine the cost of the shirt.

$$\text{One part} = 400 \div 4 = 100$$

$$\therefore \text{Cost of shirt} = 3 \text{ parts} = 3 \times 100 = 300$$

*Algebraically:*

Suppose  $x$  is the price of the shirt, then from the information provided, we can write:

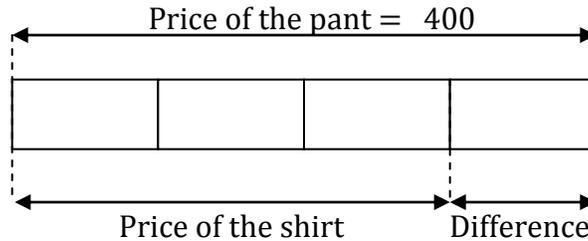
$$x = \frac{3}{4} \times \text{Price of the pant}$$

$$\therefore x = \frac{3}{4} \times 400 = 300$$

**Scenario: Given one quantity and the fraction, find the difference.**

Example:

The price of the shirt is  $\frac{3}{4}$  the price of the pant. If the pant costs Rs. 400/- then what is the difference between the price of the shirt and that of the pant?



The difference = one part =  $400 \div 4 = 100$

*Algebraically:*

Suppose  $x$  is the price of the shirt and  $y$  is the price of the pant. Now, from the information provided we can write:

$$y = 400 \text{ ----- Equation 1}$$

$$x = \frac{3}{4} \times y \text{ ----- Equation 2}$$

Substituting value of  $y$  from Equation 1 in Equation 2 we get:

$$x = \frac{3}{4} \times 400 = 300$$

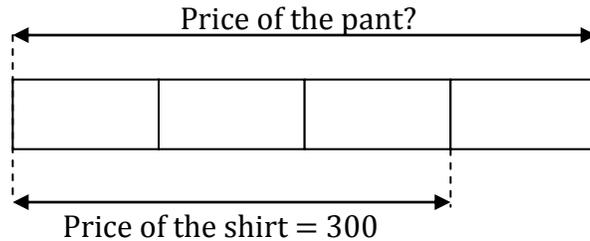
$$\therefore \text{The difference} = y - x = 400 - 300 = 100$$

So the difference between the price of pant and that of the shirt is Rs. 100/-

**Scenario: Given one quantity and a fraction find the other quantity.**

Example:

Price of the shirt is  $\frac{3}{4}$  the price of pant. If the shirt costs Rs. 300/- then what is the price of the pant?



Here “the price of the pant” is divided into 4 equal parts and of those four parts, 3 parts represent the cost of the shirt.

$$\text{One part} = 300 \div 3 = 100$$

$$\therefore \text{Cost of pant} = 4 \text{ parts} = 4 \times 100 = 400$$

*Algebraically:*

Suppose  $x$  is the price of the pant. From the information given we can write:

$$\frac{3}{4}x = 300$$

$$\therefore 3x = 1200$$

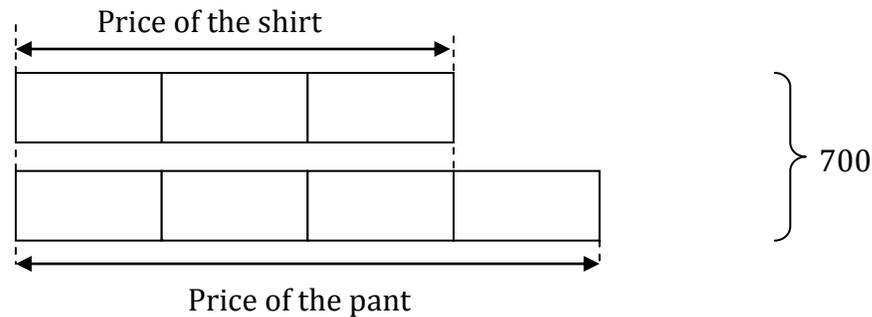
$$\therefore x = 1200 \div 3 = 400$$

So the price of the pant is Rs. 400/-

**Scenario: Given the sum and the fraction find quantities.**

Example:

Shekhar bought a pant and a shirt for Rs. 700/-. If the price of the shirt is  $\frac{3}{4}$  the price of the pant, what is the price of the pant and that of the shirt?



Here “the price of the pant” is divided into 4 equal parts and of those four parts, 3 parts represent the cost of the shirt.

$$7 \text{ parts} = 700 \therefore 1 \text{ part} = 700 \div 7 = 100$$

$$\therefore \text{Price of pant} = 4 \times 100 = 400 \text{ and that of shirt} = 3 \times 100 = 300$$

*Algebraically:*

Suppose  $x$  is the price of the shirt and  $y$  is the price of the pant. Now, from the information provided we can write:

$$x + y = 700 \text{ -----Equation 1}$$

$$x = \frac{3}{4} \times y \text{ -----Equation 2}$$

Substituting for value of  $x$  from Equation 2 in Equation 1 we get:

$$\frac{3}{4}y + y = 700$$

$$\therefore 3y + 4y = 2800 \therefore y = 2800 \div 7 = 400$$

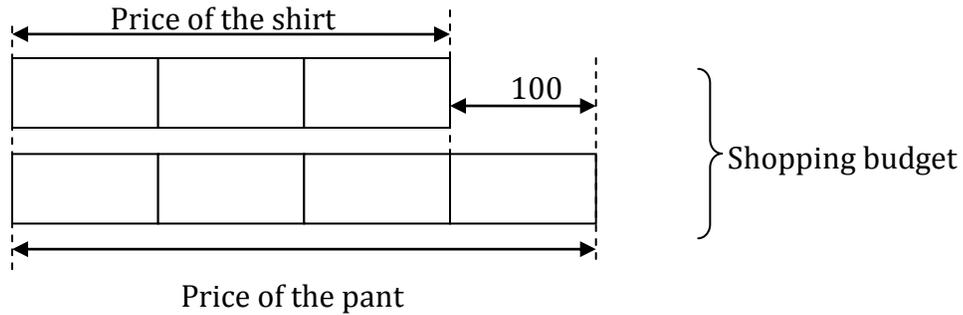
$$\therefore x = \frac{3}{4} \times 400 = 300 \text{ ---from Equation 2}$$

So the price of the pant is Rs. 400/- and that of shirt is Rs. 300/-

**Scenario: Given a fraction and difference, find quantities.**

Example:

Ulhas spent  $\frac{4}{7}$  of his shopping budget to buy a pant and the rest to buy a shirt. If the pant cost Rs. 100/- more than the shirt, find cost of the pant and that of the shirt.



$$1 \text{ part} = 100$$

$$\therefore \text{Cost of pant} = 4 \times 100 = 400 \text{ and that of shirt} = 3 \times 100 = 300$$

*Algebraically:*

Suppose  $x$  is the shopping budget, then:

$$\text{Cost of the pant} = \frac{4}{7}x \text{ and that of the shirt} = \frac{3}{7}x$$

Now, the difference between the cost of the pant and that of the shirt = 100

$$\therefore \frac{4x}{7} - \frac{3x}{7} = 100 \text{ (Multiply each term by 7)}$$

$$\therefore 4x - 3x = 700$$

$$\therefore x = 700$$

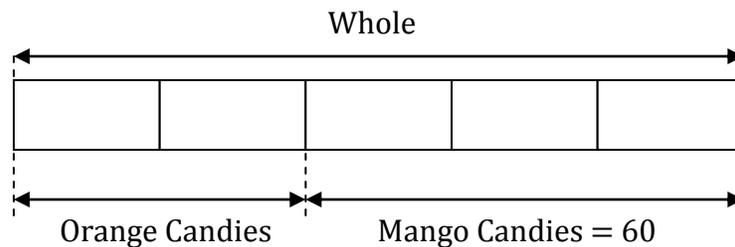
$$\therefore \text{Cost of the pant} = \frac{4}{7}x = 400 \text{ and that of the shirt} = \frac{3}{7}x = 300$$

**Scenario: Given a part and a fraction find the whole.**

Example:

Manik bought some candies.  $\frac{2}{5}$  Of the candies were Orange candies and the remaining 60 were Mango candies. So, how many candies did she buy altogether?

Here we divide the whole in 5 equal parts. Since  $\frac{2}{5}$  candies were Orange, mark 2 parts as Orange candies on the diagram, so the remaining 3 parts must be Mango candies.



Now,

$$3 \text{ parts} = 60$$

$$\therefore 1 \text{ part} = 60 \div 3 = 20$$

$$\therefore \text{Total number of candies bought} = 5 \times 20 = 100$$

*Algebraically:*

Suppose Manik bought  $x$  number of candies, since two fifth of the candies were Orange

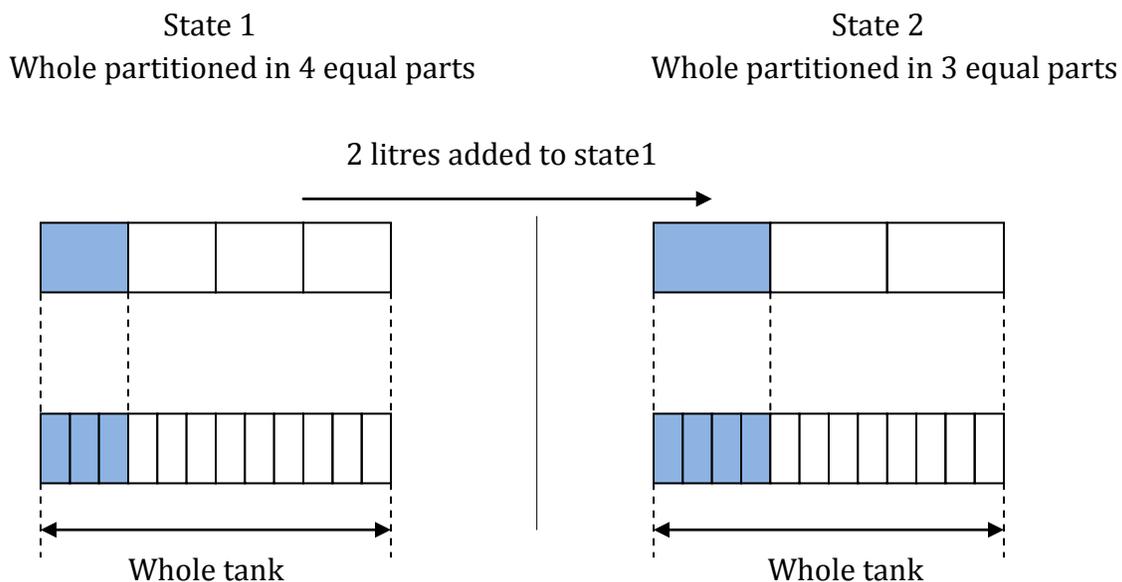
$$\begin{aligned} \text{Then Mango candies} &= x - \frac{2x}{5} = \frac{3x}{5} = 60 \\ \therefore x &= (60 \times 5) \div 3 = 100 \end{aligned}$$

$$\text{Total number of the candies bought} = 100$$

Example:

Sumedh's fish tank was  $\frac{1}{4}$  full. When he added 2 litres of water, it became  $\frac{1}{3}$  full. How much water can the tank hold if it is filled to the capacity?

Since we can see that the quantities cannot be compared easily as initially the whole is partitioned in 4 equal parts and later the whole is partitioned in 3 equal parts. It will be easier for us to make the comparison if in the initial situation we partition each part further in 3 equal parts and do the same later by partitioning each part into 4 equal parts.



From the diagram we can see that when we divide the whole in 12 equal parts one part corresponds to 2 litres of water

$$\therefore \text{The capacity of the tank} = 12 \text{ parts} = 12 \times 2 = 24 \text{ Litres}$$

*Algebraically:*

Suppose  $x$  is the capacity of the tank, from the problem we can write:

$$\frac{1}{4} x = \text{Initial quantity of water in the tank and}$$

$$\frac{1}{3} x = \text{New quantity of the water in the tank when 2 liters were added}$$

$$\therefore \frac{1}{3} x - \frac{1}{4} x = 2$$

$$\therefore (4x - 3x) / 12 = x / 12 = 2$$

(see the connection with the diagrams above?)

$$\therefore x = 24$$

**Exercise:**

1. A tank of water is  $\frac{5}{8}$  full. If it contains 200 Litres of water, what is the capacity of the tank?
2. A shop sold 15 umbrellas which were  $\frac{3}{7}$  of the original umbrellas. Find the number of umbrellas the shop had in the beginning.
3. In a box there are some yellow, red and green balls.  $\frac{1}{3}$  Of the balls are yellow. There are 30 more red balls than yellow and the remaining 15 balls are green, then how many balls are there altogether?
4. Sandhya, from her piggy bank, spent Rs. 40 on a toy and Rs.30 on a book. If she had  $\frac{2}{7}$  of her money left, how much money was there in the first place?
5. Sridhar spent  $\frac{4}{9}$  of his bonus on buying a pant and  $\frac{1}{3}$  of it buying two shirts. What fraction of the bonus remained?
6. There were 28 pens in the box.  $\frac{2}{7}$  of those pens were red and the remaining were black. How many more black pens than red pens were there in the box?
7. A school bag cost 3 times as much as a compass box. If the compass box costs Rs.55, what is a cost of the school bag and the compass box taken together?
8. Binoy had some candies. He gave  $\frac{3}{8}$  of them to Pankaj and  $\frac{2}{5}$  of the remaining to Sudarshan. If Sudarshan received 16 candies, how many candies did Binoy have in the beginning?
9. Medha and Keyur had an equal number of stamps. After Medha gave 27 stamps to Keyur, he had four times as many stamps as Medha. How many stamps did Medha have in the beginning?
10. Jyoti had some 50 paise coins and some 1 Rupee coins. The number of 1 Rupee coins is twice the number of 50 paise coins. If their total value is Rs.20/- then how many 1 Rupee coins does Jyoti have?
11. Santosh had a total of 110 mangoes and guavas. He threw away  $\frac{1}{3}$  of guavas and gave away 20 mangoes. He then found that he had equal number of mangoes and guavas left with him. How many guavas did he have in the beginning?
12.  $\frac{2}{3}$  of Mina's bangles are equal to  $\frac{3}{5}$  of Tina's bangles. If Mina has 4 bangles less than Tina, how many bangles does Tina have?
13. Hina spent  $\frac{4}{9}$  of her money on buying a sweater and  $\frac{3}{9}$  on buying a shawl. If she is left with Rs.180/- what is the price of the shawl and that of the sweater.
14.  $\frac{1}{2}$  of fruits in a basket are mangoes,  $\frac{1}{4}$  are bananas and the rest are apples. If there are 16 mangoes, then how many apples are there in the basket?

## Chapter # 5: Ratios

Ratios are a pair of numbers used to make comparison of two quantities measured in similar units. Ratios can be expressed in different ways as shown below:

2 to 3 or  $2 : 3$  or  $2/3$

Suppose in a class there are 24 girls and 26 boys.

Then the ratio of girls to boys is expressed as:

$24 : 26$  or on simplification it can be expressed as  **$12 : 13$**  or  **$12 / 13$**  or as **12 to 13**.

The ratio of boys to girls is  **$13 : 12$**  or  **$13 / 12$**  or as **13 to 12**.

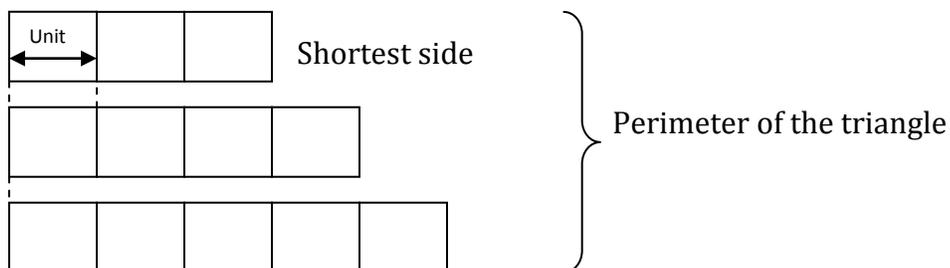
So if the same ratio is applicable to the entire school, then we can calculate number of girls in the school if we know number of boys in the school or vice versa.

Ratios can be expressed using “Part-Part-Whole Diagram” or “Comparison Diagram”

Example:

Three sides of a triangle are in the ratio  $3 : 4 : 5$ . If the shortest side of the triangle is 6 cm, what is the perimeter of the triangle?

Using “Comparison Diagram” we can express the ratio relationship between the sides of the triangle as follows:



Now, the shortest side = 3 units = 6

$$\therefore 1 \text{ unit} = 6 \div 3 = 2$$

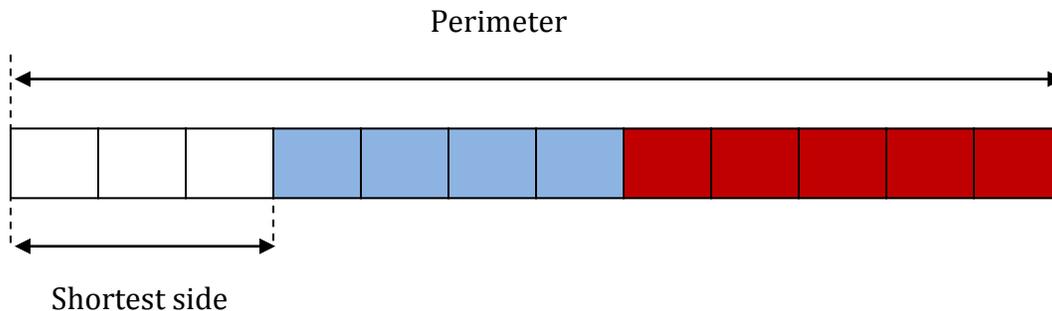
∴ Other sides of the triangle are:

$$4 \text{ units} = 4 \times 2 = 8 \text{ cm} \quad \text{and} \quad 5 \text{ units} = 5 \times 2 = 10 \text{ cm}$$

$$\text{Perimeter of the triangle} = 5 + 4 + 3 = 12 \text{ units} = 12 \times 2 = 24 \text{ cm}$$

The above problem can also be solved using “Part-Part-Whole Diagram”.

Here, the perimeter is the “Whole” and the sides are the “Parts”.



$$\text{Shortest side} = 3 \text{ units} = 6 \text{ cm}$$

$$\therefore 1 \text{ unit} = 6 \div 3 = 2$$

∴ Other sides of the triangle are:

$$4 \text{ units} = 4 \times 2 = 8 \text{ cm} \quad \text{and} \quad 5 \text{ units} = 5 \times 2 = 10 \text{ cm}$$

$$\text{Perimeter of the triangle} = 5 + 4 + 3 = 12 \text{ units or } 12 \times 2 = 24 \text{ cm}$$

*Algebraically:*

Let  $x$  be the unit of measure for this ratio. Then we can express the ratio as:

$$3x : 4x : 5x$$

$$\text{Now, the shortest side} = 3x = 6$$

$$\therefore x = 6 \div 3 = 2 \text{ cm}$$

$$\text{And the perimeter of the triangle} = 3x + 4x + 5x = 12x = 12 \times 2 = 24 \text{ cm}$$

Alternately:

Let  $x, y, z$  be the three lengths of the triangle and let  $x$  be the shortest length. Then

$$x/y = 3/4$$

$$\therefore 4x = 3y$$

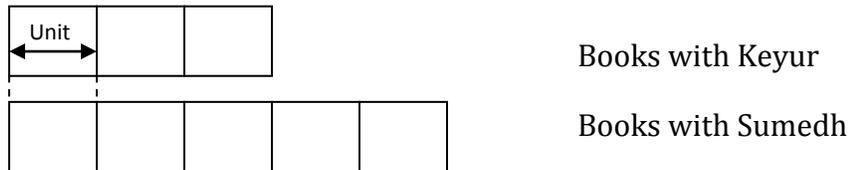
$$\therefore 4 \times 6 = 3y$$

$$\therefore y = 8$$

**Scenario: Ratio between two quantities and one of the quantities is given. We have to determine the other quantity.**

Example:

Keyur and Sumedh have books in the ratio 3 : 5. If Sumedh has 30 books, how many books does Keyur have?



Since Sumedh has 30 books,

$$5 \text{ units} = 30$$

$$\therefore 1 \text{ unit} = 30 \div 5 = 6$$

$$\therefore \text{Number of books with Keyur} = 6 \times 3 \text{ units} = 18$$

*Algebraically:*

Let  $x$  be the unit of measure for this ratio. Then we can express the ratio as:

$$3x : 5x$$

$$\text{Now the books with Sumedh} = 5x = 30$$

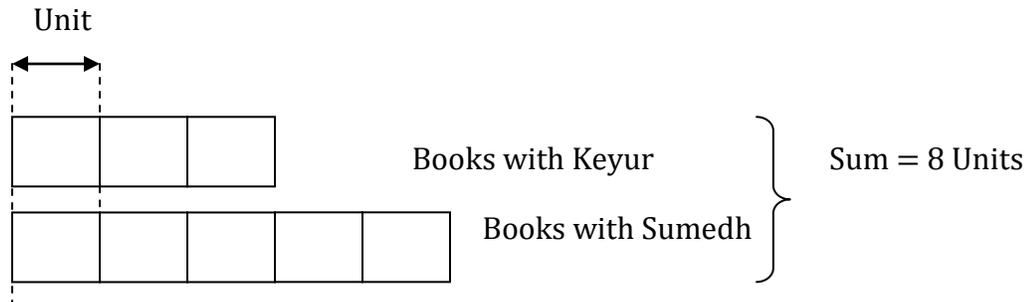
$$\therefore x = 30 \div 5 = 6$$

$$\therefore \text{Books with Keyur} = 3x = 3 \times 6 = 18$$

**Scenario: Ratio between two quantities and one of the quantities is given. We have to determine the sum of the two quantities.**

Example:

Keyur and Sumedh have books in the ratio 3 : 5. If Sumedh has 30 books, how many books they have altogether?



Since Sumedh has 30 books,  $5 \text{ units} = 30$

$$\therefore 1 \text{ unit} = 30 \div 5 = 6$$

Sum of the books with Keyur and Sumedh = 8 units =  $8 \times 6 = 48$

*Algebraically:*

Let  $x$  be the unit of measure for this ratio. Then we can express the ratio as:

$$3x : 5x$$

Now, the books with Sumedh =  $5x = 30$

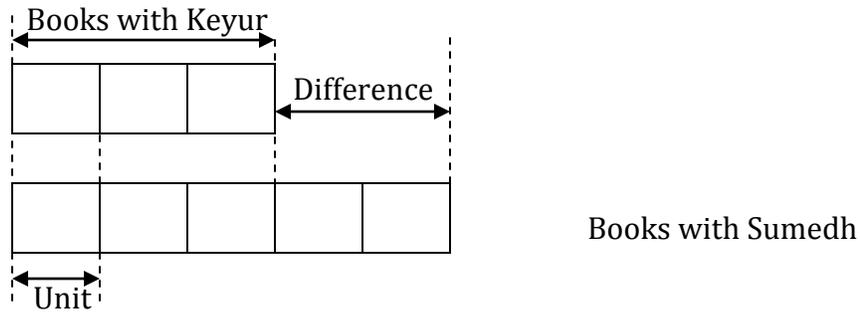
$$\therefore x = 30 \div 5 = 6$$

$\therefore$  Books with Keyur & Sumedh =  $8x = 8 \times 6 = 48$

**Scenario: Ratio between two quantities and one of the quantities is given. We have to determine the difference between the two quantities.**

Example:

Keyur and Sumedh have books in the ratio 3 : 5. If Sumedh has 30 books, how many more books does Sumedh have than Keyur?



Since Sumedh has 30 books,

$$5 \text{ units} = 30$$

$$\therefore 1 \text{ unit} = 30 \div 5 = 6$$

$$\text{The difference} = 2 \text{ units} = 2 \times 6 = 12$$

$\therefore$  Sumedh has 12 more books than Keyur

*Algebraically:*

Let  $x$  be the unit of measure for this ratio. Then we can express the ratio as:

$$3x : 5x$$

$$\text{Now, the books with Sumedh} = 5x = 30$$

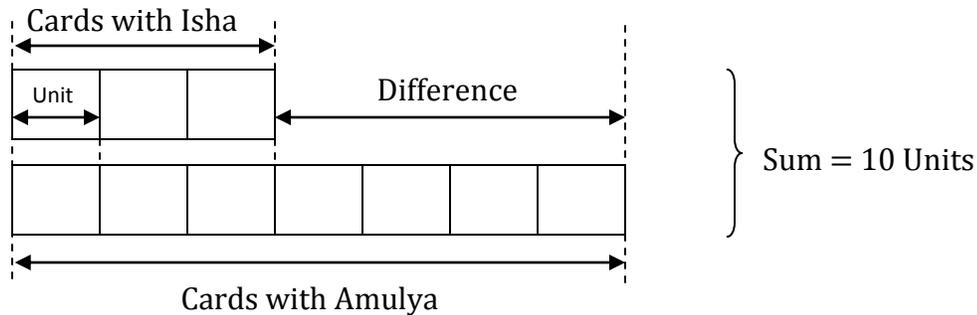
$$\therefore x = 30 \div 5 = 6$$

$$\therefore \text{The difference} = 5x - 3x = 2x = 2 \times 6 = 12$$

**Scenario: Ratio between two quantities and the difference is given. We have to determine a quantity.**

Example:

Amulya and Isha shared some game cards in the ratio of 7 : 3. When Amulya gave Isha 16 game cards, she found that they each have the same number of cards. How many cards did Amulya have in the beginning?



Amulya and Isha together have  $7 + 3 = 10$  Units

For Amulya and Isha to have equal cards, each must have 5 units

Since, after Amulya giving 16 cards to Isha, they had equal number of cards.

That means Amulya must have given Isha cards corresponding to 2 units

$$\therefore 2 \text{ unit} = 16$$

$$\therefore 1 \text{ unit} = 16 \div 2 = 8$$

$$\therefore \text{In the beginning, cards with Amulya} = 7 \text{ units} = 7 \times 8 = 56$$

*Algebraically:*

Let  $x$  be the unit of measure for this ratio. Then we can express the ratio as:

$$3x : 7x$$

$$\text{Cards with Isha} = 3x \text{ and Cards with Amulya} = 7x$$

Now we know that:

$$3x + 16 = 7x - 16$$

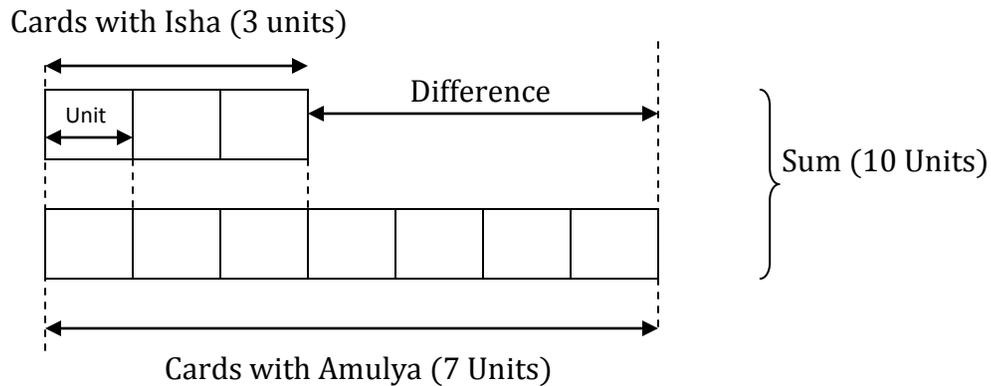
$$\therefore 4x = 32 \Rightarrow \therefore x = 8$$

$$\therefore \text{In the beginning, cards with Amulya} = 7x = 7 \times 8 = 56$$

**Scenario: Ratio between two quantities and the difference is given. We have to determine the sum of quantities.**

Example:

Amulya and Isha shared some game cards in the ratio of 7 : 3. Amulya has 32 more cards than Isha. How many cards did Amulya and Isha have in the beginning all together?



=

$$\text{The difference} = 4 \text{ units} = 32$$

$$\therefore 1 \text{ unit} = 32 \div 4 = 8$$

$$\therefore \text{Cards with Amulya and Isha} = 10 \text{ units} = 10 \times 8 = 80$$

*Algebraically:*

Let  $x$  be the unit of measure for this ratio. Then we can express the ratio as:

$$3x : 7x$$

$$\text{Cards with Isha} = 3x \text{ and Cards with Amulya} = 7x$$

Now we know that:

$$7x - 3x = 32$$

$$\therefore 4x = 32 \quad \therefore x = 8$$

$\therefore$  In the beginning,

$$\text{Cards with Amulya and Isha taken together} = 7x + 3x = 10x$$

$$\therefore 10x = 10 \times 8 = 80$$

**Exercise:**

1. The ratio of two numbers is 5 to 2. If the sum of the two numbers is 12 more than the difference, find the numbers.
2. Kaustubh and Ketki made some paper boats in the ratio 5 : 3. Kaustubh gave half of his paper boats to Ketki. Ketki then had 24 more boats than Kaustubh. How many paper boats did they have altogether?
3. In a game, Varun and Venkat shared some cards in the ratio 5 : 4. Venkat then lost half his cards to Varun. Varun then had 35 cards. So, how many cards did they have altogether?
4. Manoj and Saurabh shared Rs.350/- in the ratio 1 : 4. Each spent half of his share on books. How much more money did Saurabh have compared to Manoj?
5. Men, women and children attended a program in the ratio of 5 : 4 : 2. If there were 36 more women than children, how many men attended the program?
6. At a party, the ratio of the number of boys to the number of girls is 1 : 3. If each of the girls is given 3 booklets and each of the boys is given 4 booklets, a total of 234 booklets will be needed. Then how many children are there at the party?
7. Partha keeps his marbles in two boxes. There are twice as many marbles in box 2 as in box 1. Box 1 contains all green marbles and box 2 contains green marbles and yellow marbles in the ratio 3 : 4. If there are 78 green marbles in all, how many yellow marbles are there?
8. Amulya and Vina each have some money. If Amulya spends Rs.4/- the ratio of money that she has with the money that Vina has will be 3 : 5. If Vina spends Rs.4/- the ratio of money that Amulya has to the amount of money that Vina has will be 11 : 13. How much money does each girl have?
9. The ratio of Aishwarya's age and Sushmita's age is 5 : 6. If their total age is 66, how old is Aishwarya?
10. On a farm the ratio of the number of chickens to the number of goats is 7 : 3. If there are 40 more chickens than the goats then how many goats are there on the farm?
11. Trishna had 150 stamps. She shared half of her stamps with her friends Tanvi and Tanuja in the ratio 2 : 3. So how many stamps did Tanvi receive?
12. Box 1 and box 2 both contained some black and blue balls. The ratio of black balls to blue balls was 3 : 2 in box 1 and was 1 : 2 in box 2. Box 1 contains three times as many balls as box 2. If box 1 has 135 balls then what is the ratio of black balls in box 1 to blue balls in box 2.
13. A piece of string 72 cm long was cut into three pieces. The length of the three pieces was in the ratio 2 : 3 : 4. What was the length of the shortest piece?
14. In a robotics class the ratio of the number of boys to the number of girls is 9 : 4. If there are 65 students in the class. How many boys are there in the class?

15. In a fruit basket there are apples, mangoes and pomegranates in the ratio  $4 : 3 : 2$ . If there are 6 pomegranates, how many fruits are there altogether?

## Chapter # 6: Percentage

Percent - or per hundred, or out of hundred - is a special case of ratios or fractions where the second term or the denominator is 100. Percentages are expressed with the % sign. We can express a ratio or a fraction as a percent and vice versa. It is important to understand what forms the base of our percent scale.

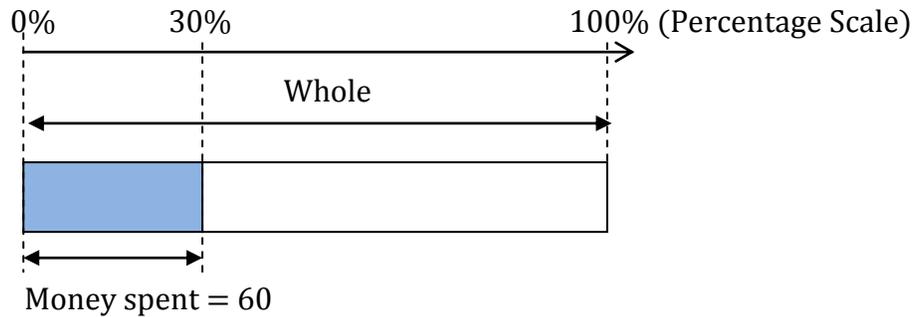
We can apply the “Part-Part-Whole” or “Comparison” Diagrams to problems related to percentages just like ratios, or fractions.

**Scenario: Given a part and percent, calculate the whole.**

Example:

Kavita bought a toy for Rs. 60/-. If she used 30% of the money in her piggy-bank to buy the toy, how much money was there in her piggy-bank before she bought the toy?

We can represent all this using our “Part-Part-Whole” Diagram:



Here the whole is: The amount available with Kavita =?

This whole is divided into 100 parts.

Each part corresponds to 1% of the whole.

Kavita spent Rs.60/- which is 30% of the whole; this means 1% of the whole =  $60 \div 30 = 2$

The money available in her piggy-bank was:

$$100 \times 2 = 200$$

So, Kavita had Rs. 200/- in her piggy-bank.

*Algebraically:*

Let Kavita have  $x$  amount in her piggy bank. From the information given we can write:

$$(30/100)x = 60$$

$$\therefore x = 6000 \div 30 = 200$$

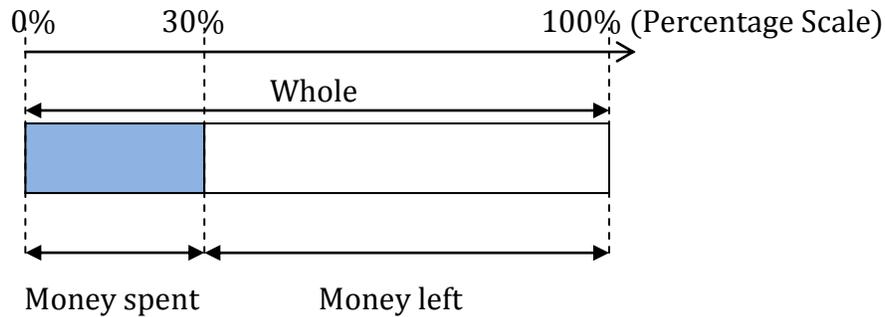
So Kavita had Rs.200/-in her piggy bank.

**Scenario: Given a part and percent calculate the other part.**

Example:

Kavita bought a toy for Rs. 60/-. If she used 30% of the money, in her piggy-bank, to buy the toy, how much money was left in her piggy-bank?

We can represent all this using our “Part-Part-Whole” Diagram.



This whole is divided into 100 parts; each part corresponds to 1% of the whole.

Now, 30% or 30 parts of the whole were spent for buying the toy.

This means money remaining in her piggy-bank is 70 percent or 70 parts.

Kavita spent Rs.60/- which is 30% of the whole, this means:

$$1\% \text{ of the whole} = 60 \div 30 = 2$$

The money remaining in her piggy-bank was:

$$70 \times 2 = 140,$$

So, money remaining in her piggy-bank is Rs. 140/-

*Algebraically:*

Let Kavita have  $x$  amount in her piggy bank. From the information given we can write:

$$(30/100)x = 60$$

$$\therefore x = 6000 \div 30 = 200$$

$$\therefore \text{Money left} = (70/100) x = 140$$

So Kavita had Rs.140/- left in her piggy bank.

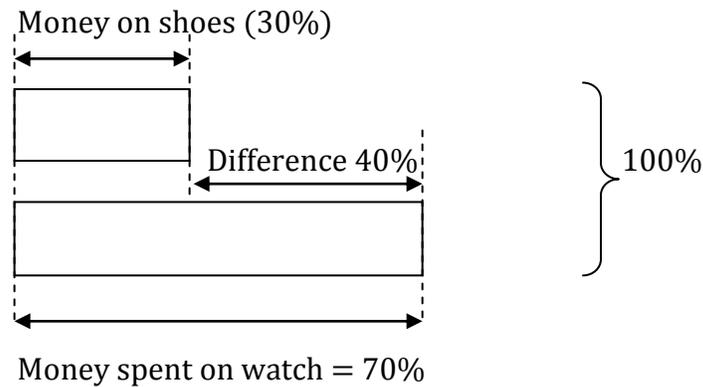
**Scenario: Given a part, percent and the difference; calculate the whole.**

Example:

Ravi bought a watch and a pair of shoes. He spent 30% of the money on the shoes & the watch cost Rs. 400/- more than the shoes. How much money did Ravi spend altogether?

Money spent on watch + Money spent on shoes = Total money spent (“The Whole”)

If the whole is divided into 100 parts; each part corresponds to 1% of the whole.



We can see that  $40\% = 400$

$$\therefore 1\% = 10$$

$$\therefore \text{Total money spent} = 100\% = 100 \times 10 = \text{Rs.}1000/-$$

*Algebraically:*

Suppose  $x$  is the money spent,

$$\text{Money spent on shoes} = \left(\frac{30}{100}\right) x$$

$$\therefore \text{Money spent on watch} = \left(\frac{70}{100}\right) x$$

Since the watch cost Rs.400/- more than the shoes,

$$\frac{70x - 30x}{100} = 400$$

$$40x = 40000$$

$$\therefore x = 1000$$

Total money spent equals Rs. 1000/-.

**Scenario: Given the whole and percent calculate the part.**

Example:

Shalva bought a saree; the shopkeeper gave her a 20% discount on the original price. If the original price of the saree was Rs. 1800/-, how much money did Shalva pay for the saree?



Since the discount is on the original price, we divide the original price into 100 parts.

$$\therefore 1 \text{ part or } 1\% = 1800 \div 100 = 18$$

Money paid is 80% of the original price.

$$\therefore \text{Money paid} = 80 \times 18 = 1440$$

So the money paid for the saree was Rs. 1440/-

*Algebraically:*

Suppose  $x$  is the money paid by Shalva. Then:

$$x = (80/100) \times \text{the original price}$$

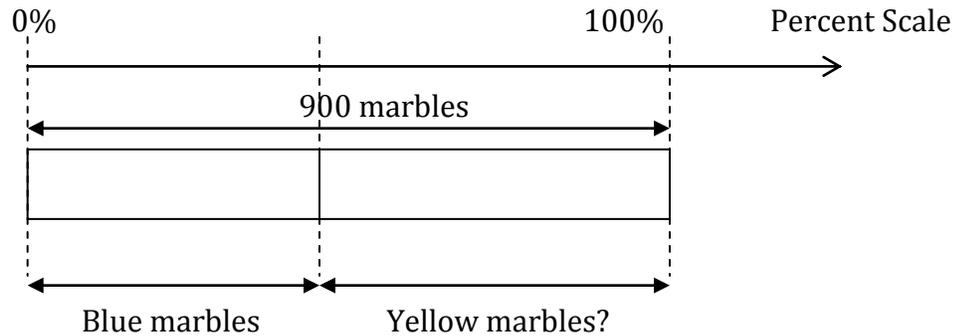
$$\therefore x = \frac{80 \times 1800}{100} = 1440$$

So the money paid for the saree is equal to Rs. 1440/-

**Scenario: Given a whole and a part calculate other part in percent**

Example:

Sujay has 900 blue and yellow marbles all together. If he has 405 blue marbles, what percentage of the marbles are yellow?



Here we apply percent scale to total number of marbles.

Divide 900 marbles in 100 parts, each part corresponds to 1%

$$\therefore 1\% = 900 \div 100 = 9$$

$$\begin{aligned} \text{Number of yellow marbles} &= \text{Total marbles} - \text{Blue marbles} \\ &= 900 - 405 = 495 \end{aligned}$$

$$\therefore \text{Yellow marbles as a percent of all the marbles} = 495 \div 9 = 55\%$$

*Algebraically:*

Suppose  $x\%$  marbles are yellow.

Then  $(100 - x)\%$  marbles are blue

$$\text{Now percentage of blue marbles is} = \left(\frac{405}{900}\right) \times 100 = 45\%$$

$$\therefore 100 - x = 45$$

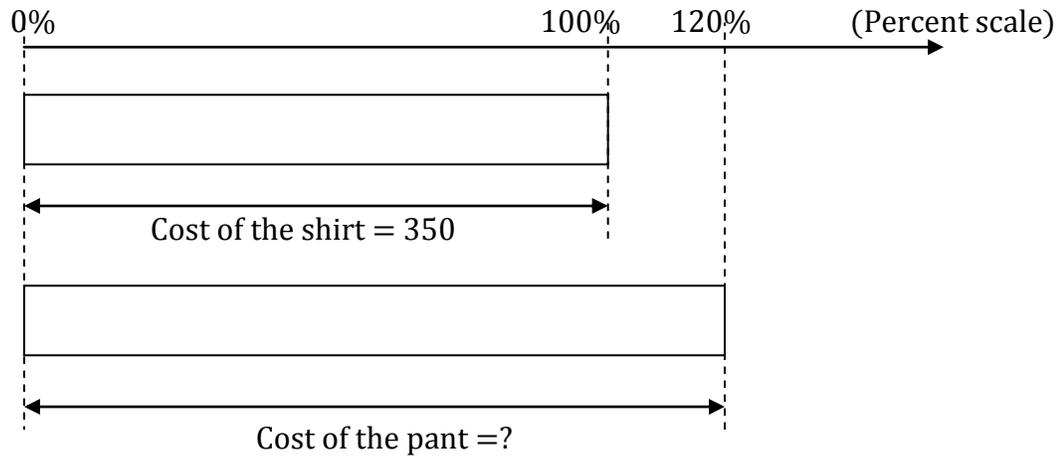
$$\therefore x = 55$$

**Scenario: Given a quantity and a relation in percent, calculate the other quantity.**

Example:

Price of a shirt is Rs. 350/-. If price of a pant is 20% more than that of the shirt, how much does the pant cost?

A “Comparison” Diagram for the problem will look like this:



Since the price of the pant is expressed in relation with that of the shirt, apply the percentage scale with the price of the shirt as 100% or 100 parts.

$$\therefore 100 \text{ parts} = 350$$

$$\therefore 1 \text{ part} = 350 / 100 = 3.5$$

$$\therefore 120 \text{ parts} = 120 \times 3.5 = 420$$

The price of the pant is Rs. 420/-

*Algebraically:*

Let us suppose  $x$  is the price of the pant. Then:

$$x = 350 + (20/100) \times 350$$

$$\therefore x = 420$$

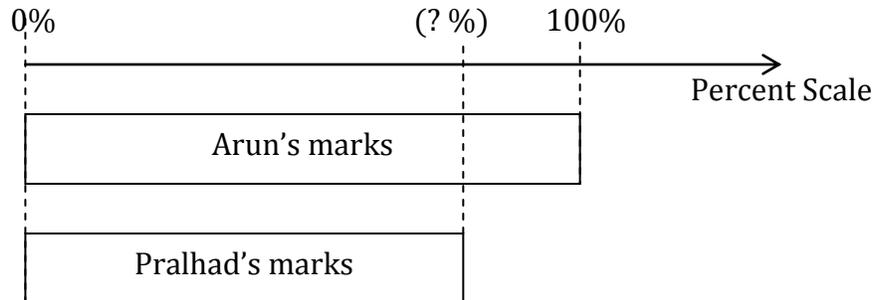
So, the price of the pant is Rs. 420/-

**Scenario: Given two quantities, compare them using percent.**

Example:

In a mathematics exam Arun scored 95 marks out of 100 and Pralhad scored 76 out of 100. Express marks scored by Pralhad as a percentage of marks scored by Arun.

Since we have to express marks scored by Pralhad in terms of marks scored by Arun, we take marks scored by Arun as the base for the percent scale



$$100\% \text{ or } 100 \text{ parts} = 95$$

$$\therefore 1 \text{ part} = 95/100$$

$$\text{Pralhad's marks correspond to } 76 \div (95/100) = \left(\frac{76 \times 100}{95}\right)\%$$

$\therefore$  Pralhad's score corresponds to 80% of what Arun scored.

*Algebraically:*

Suppose Pralhad's score corresponds to  $x\%$  of marks scored by Arun. Then,

$$76 = (x / 100) \times 95$$

$$\therefore x = \left(\frac{76 \times 100}{95}\right)\% = 80\%$$

$\therefore$  Pralhad's score corresponds to 80% of what Arun scored.

### Exercise:

1. A book was sold at a discount for Rs. 320/-. If its original price was Rs. 400/-, find, in percent, how much the book was discounted with reference to the original price. If you have to take the price back to its original level, by how many percent should you increase the price?
2. There were 80 children in the bus and 30% of them were boys. When some boys got off the bus, the number of the boys on the bus became 20%. How many boys got off the bus?
3. A toy was sold for Rs. 24/-. If the price was 20% less than the original price, what was the original price?
4. Siddhartha has 80% more picture cards than Atharva. Makrand has 15% less picture cards than Siddhartha. If the difference in the number of picture cards that Makrand and Atharva have is 106, how many picture cards does Atharva have?
5. In a class, 40% students are boys. If, from that class, 20% of the boys and 30% girls are participating in a chess competition, how many students from that class are participating in chess competition? Give your answer in percent.
6. Akki had 120 marbles. 20% of these were blue and the rest were yellow. Then, in a game, he won some more blue marbles and now the blue marbles were 40% of the total marbles. Find out how many blue marbles Akki won.
7. In a school library there are 2400 books. 30% of these are English books, 480 books are in Hindi and the rest are in Marathi. Find the number & percentage of books in Marathi.
8. Isha read quarter of a book on Monday and 80% of the remaining book on Tuesday. On Wednesday, she finished the book by reading the last 30 pages. Find the total number of pages in the book.
9. In a program there are 360 more boys than the girls. If the number of girls is 40% of the total number of children, how many children are there?
10. 40% of number A is 50% of number B. If the difference between the numbers is 15, find the value of both the numbers.
11. At a program 30% of the participants were children. The number of men was 20% more than the number of children. If there were 272 women at the program, how many people attended the program?
12. Farmer Dhondu distributed some saplings among his sons. First one got 30% and the second son got 20% of the total saplings. If the third son got 200 fewer saplings than the first son and the fourth son got 600 saplings, how many saplings were there all together?
13. A tank of water was 30% full. When 300 liters of water was added it became 45% full. What is the capacity of the tank?

14. In a box there are some beads. 35% are red beads and the rest are green beads. If there are 450 more green beads than red, then how many beads are there altogether?
15. Alhad, Akshata and Atharva shared some stickers. Alhad received 20% of the stickers and Akshata received 65% of the remaining stickers. If she received 48 stickers more than Atharva, then how many stickers were there altogether?

Taking from Jean Piaget, we consider children as builders of their own intellectual structures. We at **Protean Knowledge Solutions** just provide the necessary inputs.

## FAQs

Is Diagrammatic Representation the only method available for solving word problems?

No. There are many methods we can use to solve a word problem. Some problems may need completely different approach like - make a list, work backwards, guess and check, look for pattern... etc. However, as shown in this booklet, many problems can be solved using simple diagrams.

When a student should stop using Diagrammatic Representation and use algebra instead?

Diagrammatic Representation is only a tool or an intermediate step. Using algebraic expressions to solve a problem is our ultimate goal. As such, irrespective of the grade, once a student becomes comfortable in interpreting information given in the problem using algebraic expressions, he or she can stop use of Diagrammatic Representation.

Do we have to construct diagrams to solve all the problems in this book?

No. Some problems are really nasty and don't break your head to solve them using neat and nice diagrams. Instead, use diagram to make the idea clear and then use algebraic expressions.

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