Fun with Tiles: A Math-Art Activity Book

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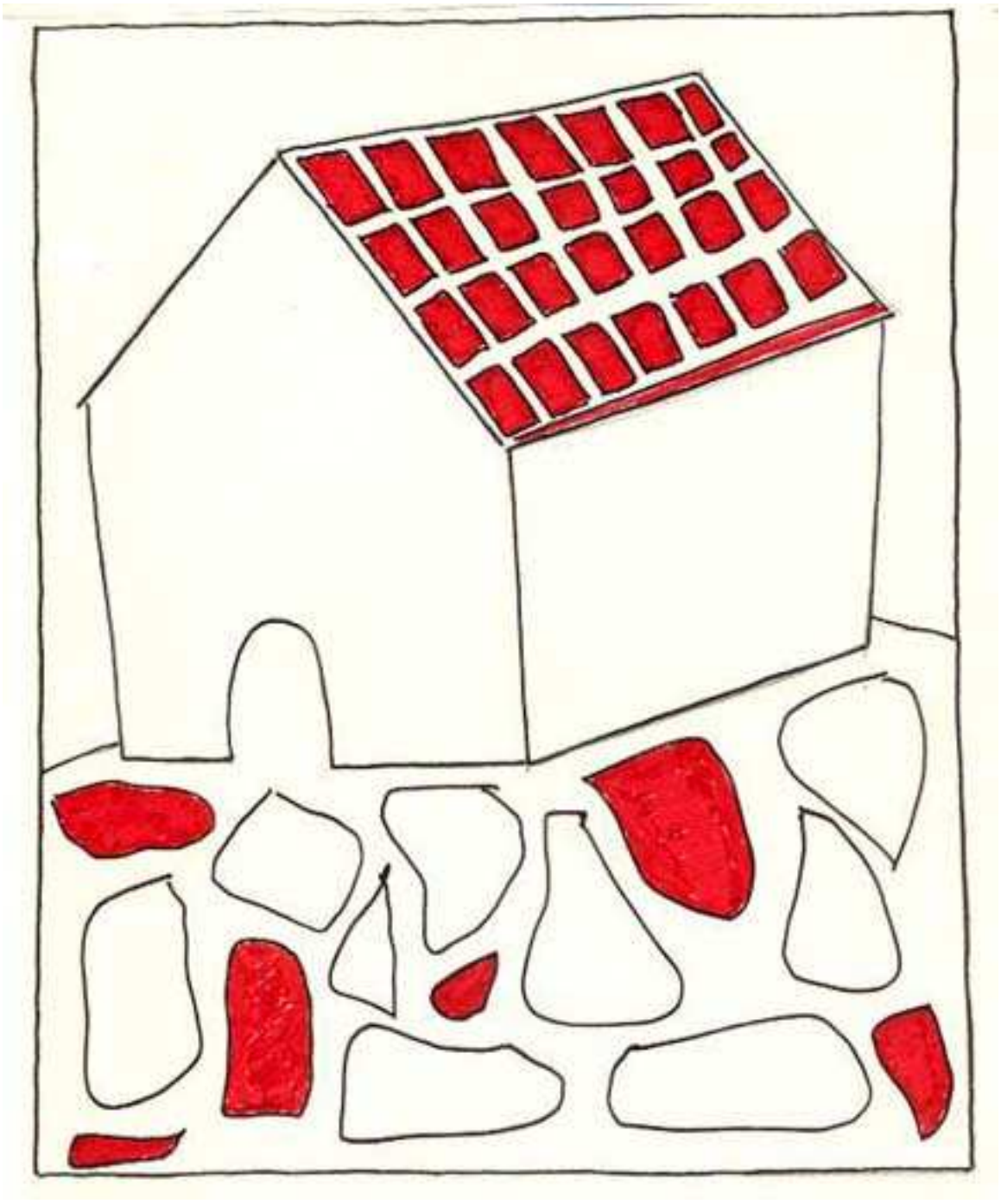


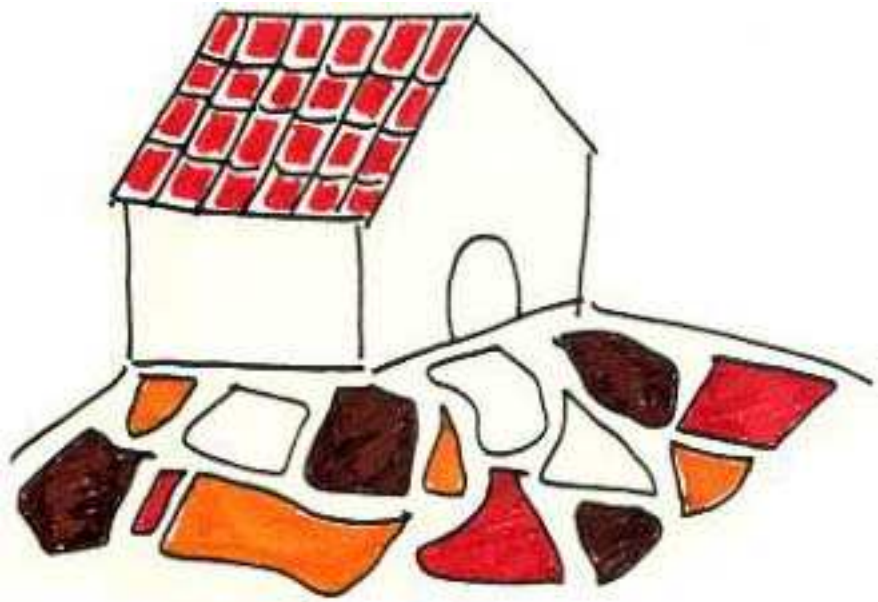
Tiling is a deeply fascinating area of mathematics. It has interested not only mathematicians, but also artists and architects. Even today tiling is an active

area of research in modern mathematics. At the same time, certain visual aspects of this problem are accessible to children.
In this book we bring out the excitement of the mathematics and art of tiling through a few math-art activities. The examples given are derived from arts and crafts all over India. This is an attempt at making mathematics enjoyable through its application in arts and crafts. Children will develop a concrete understanding of the concepts in geometry and symmetry by using them in design. This book is not meant to be part of a conventional exam and syllabus oriented school curriculum. It is designed to excite children who are especially interested in mathematics or art or both.

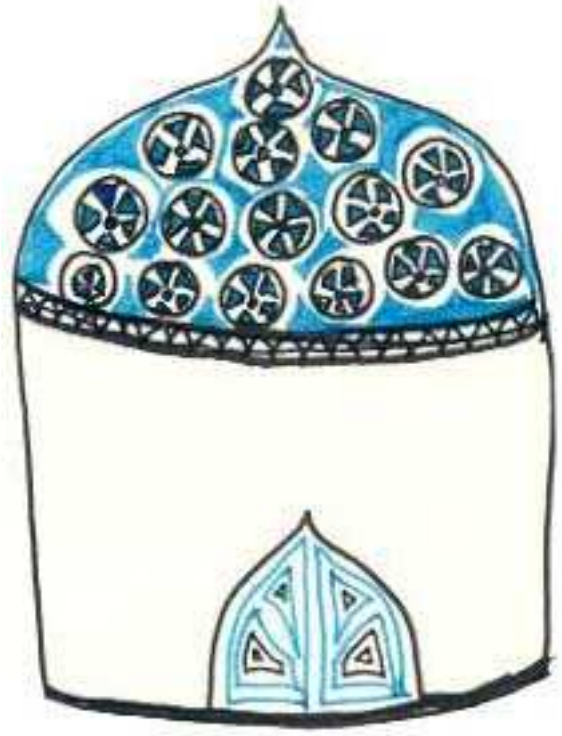
Contents

1. Introduction to Tiling

We see tiles all around us: on roofs, on floors, in courtyards, on bathroom walls. Tiles are used to cover surfaces. These tiles do not overlap and there are no gaps between them.



Sometimes the tiles are irregular like the ones shown on the courtyard. More often, we see identical tiles covering a plane – like the ones on roofs or on our bathroom walls. These are easier to make in a factory – you just keep making the same tiles over and over again!
These identical tiles are then used to cover a surface in a repeating pattern. We don't want our bathroom walls to get wet, so we don't leave any gaps between the tiles.



If we look harder we see many, many more examples of repeating patterns all around us.

On woven mats

On block printed
Bed spreads

On jute
Bags

On Kameezes

In this book we deal with regular, repetitive tiling.

2. Making a kite – fun with regular polygons

Things you need:
1. A white sheet of paper
2. A few sheets of coloured marble paper
3. A cardboard sheet
4. A pair of scissors
5. Glue
6. A ruler
7. A pencil

- Cut out a square sheet of white paper.

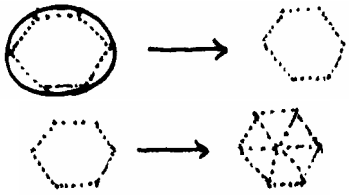
- To decorate this square shaped paper kite we will use bits of colour paper shaped like regular polygons.
- Cut out coloured bits of paper shaped like equilateral triangles, squares, rectangles, pentagons and hexagons. Use cardboard stencils of these shapes to make these bits (see next page)
- Now decorate the square sheet of white paper by covering it up with coloured bits of paper of the same shape and size. For instance, by

covering it up with square bits of paper we get a design like this: So, at the end we get a lovely kite!

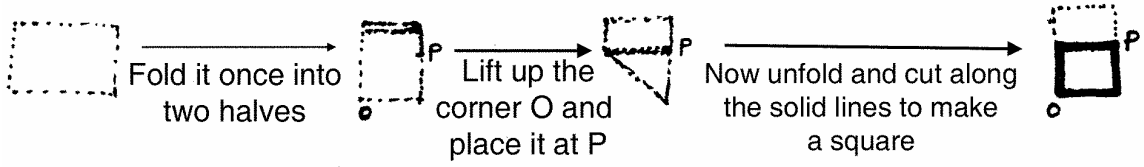
- Did you manage to fill up the kite without leaving gaps by repeating every one of the polygonal shapes? If not, do you understand why tiling works only for some polygons: not all?

Here is how you make cardboard stencils of regular polygons (if you have trouble with this, ask your teacher or parent to help you.)

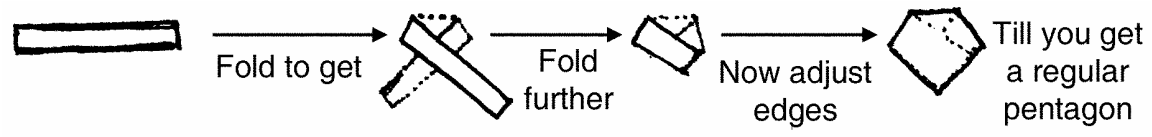
1. A regular hexagon: Take a circular bottle cap of suitable size and cut out two circles out of a piece of cardboard. Cut one of the circles into two equal parts by folding and overlaying one half of the circle onto the other half. Now place the centre of one of these semi-circular cardboard pieces on the circumference of the other circle and divide this circumference into six equal parts. Join these points with a pencil and a ruler to form a regular hexagon. Cut out the circular cardboard into this regular hexagon shape to use it as a template. Make two such templates.
2. An equilateral triangle: Cut out the second hexagonal template into six equal triangles. One of these six triangles can serve as an equilateral triangle template.



3. A square: Take a rectangular sheet of paper. Make the following moves to make a square out of it.



4. A pentagon: Take a strip of paper and do the following moves:

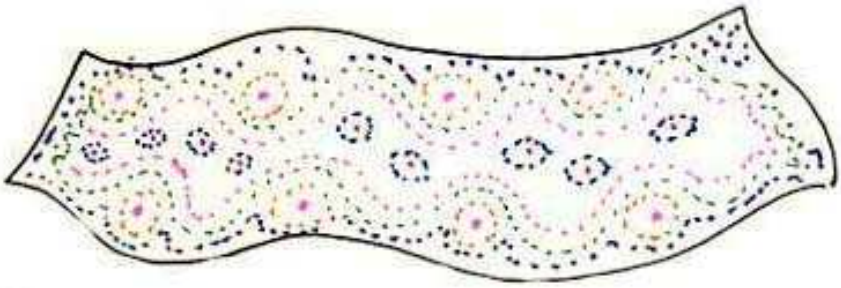


When you make these regular polygonal templates you need to choose the size of these units in such a way that you can use a fairly large number of them to cover the kite.

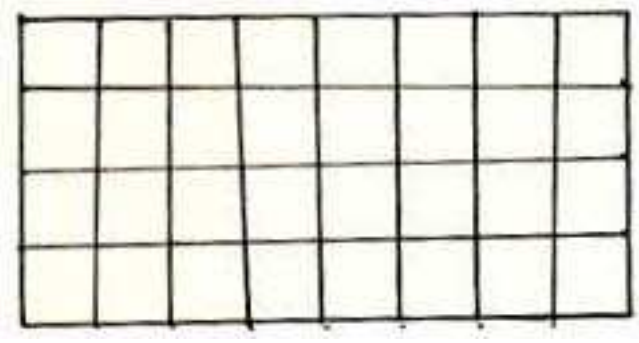
Designing a Quilt ("Kantha")

Things you need:

1. A white sheet of paper
2. A cardboard sheet
3. A pair of scissors.
4. A ruler
5. A pencil
6. A set of sketch pens

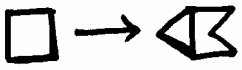


In Bengal there is a tradition of making beautifully embroidered quilts called "Kanthas'. These Kantha stitches (running stitches of coloured threads) are also used on sarees and shawls. In activity first you gained some experience with handling regular polygons.



Now let us take these basic regular polygons and make them look more interesting. We can use these to design Kanthas. We will draw these designs on paper. If any of you are good at stitching you can actively try it out on a piece of cloth with a needle and coloured thread. Here is an example:

Start with a square tiling pattern. Now change each polygon unit (Tile) at the edges as follows:

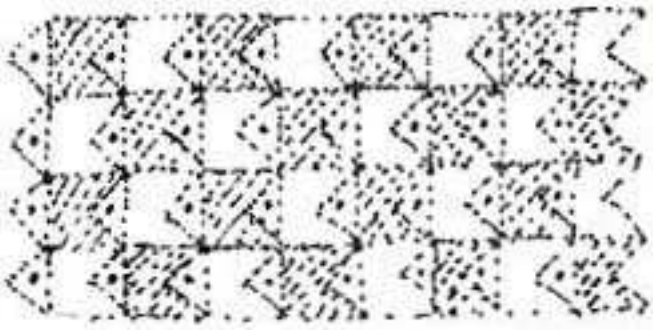


What we have done is to cut a triangle near one edge and paste it to the edge parallel to it.

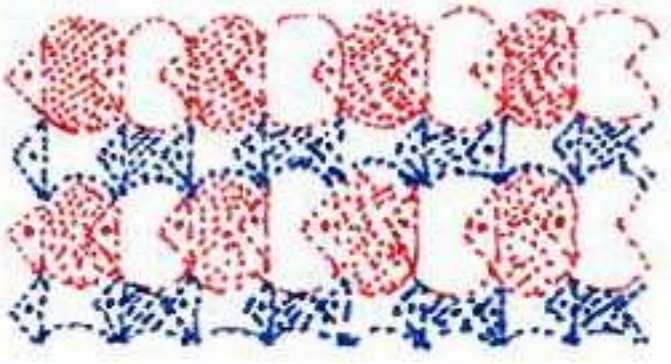
Notice that this fish like shape 

can be rearranged to look like a square. 

You can check this for yourself using a piece of cardboard. With this basic fish-shaped tile we can now tile the rectangle, but now it looks a lot more interesting.



We can even change it a bit to make a design of fat and thin fish. Can you figure out how I have gone from the earlier design to this new one?



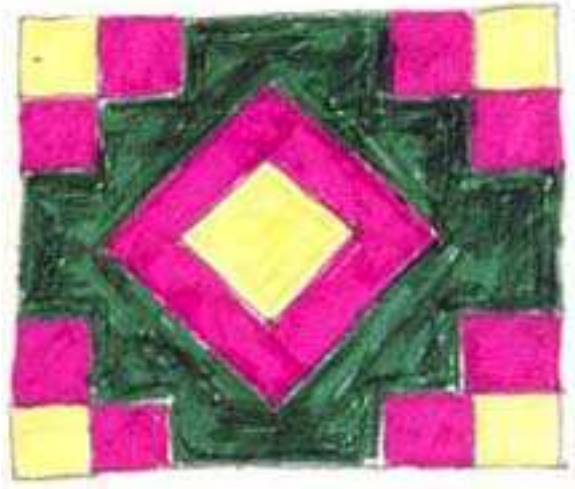
Now change the other regular polygonal repetitive pattern you made in Activity I and make your own lovely Kantha designs. In each case, keep track of the moves you make to change the regular polygonal units into other interesting patterns.

Make sure that when you reassemble the parts that make the changed shape, you get back the original polygonal shape.

Drawing a Rangoli

Things you need:

1. A white sheet of paper
2. A cardboard sheet
3. A pair of scissors.
4. A ruler
5. A pencil
6. A set of sketch pens



Rangolis are very common. We often draw them on our courtyards. Festivals like Diwali are celebrated with beautiful Rangoli Patterns. A typical Rangoli pattern is shown here. Notice a few interesting features of this picture. If you pin down the centre of the design with your finger and turn it around, you find that for certain angles of turning, the picture looks the same. Can you check for yourself which angles of turning keeps the pictures looking the same? You can also think of the picture as made up of two halves. You can draw one half and the other half looks like a mirror image of the first half. Can you try these out yourself?

Now let us make a Rangoli of our own. I will give you one example. Let us start with a regular pattern of up and down triangles. Now let us change each up triangular block  into 

and each down triangular block  into 

Notice that you fit the two new blocks together to get 

which can be exactly overlaid onto 

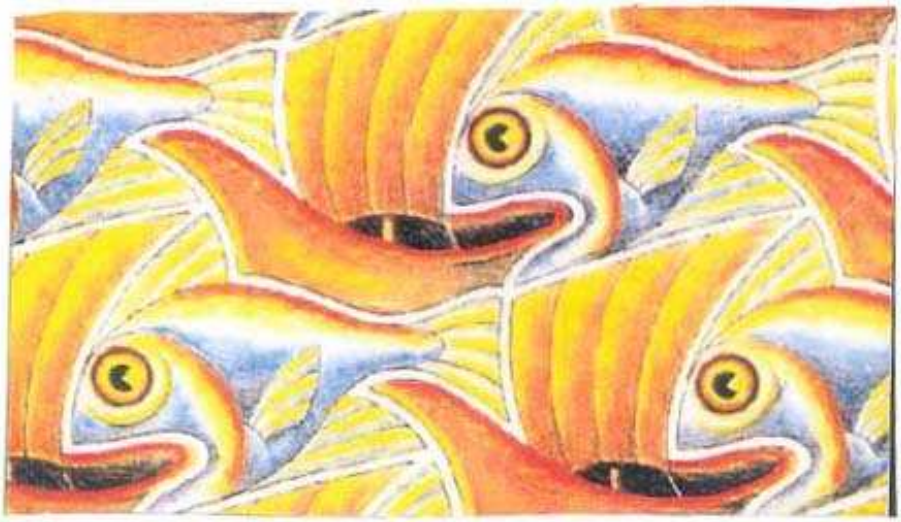
a combined block of up and down triangles. Check this by making cardboard stencils of these blocks.

Now by repeating these two new blocks you can make a beautiful Rangoli pattern like the one I have shown you here.

When you make a Rangoli starting from a regular polygonal pattern make sure that you get the original repeating block when you reassemble the parts of the new repeating block. Try making the designs on a piece of paper before trying them out on floors with chalk and coloured powder.

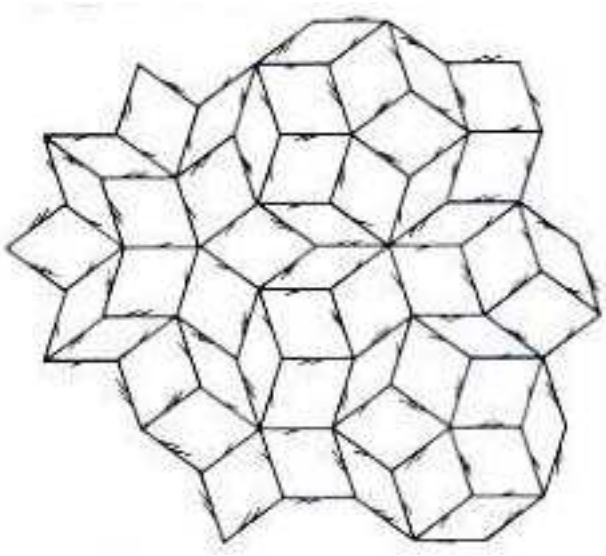
In each of these Rangoli patterns try to look for moves which make the pattern look the same as the original picture. For instance, in this picture 'Rangoli of dancers' we can see that one half of the picture looks like a mirror image of the other half. Can you see any other moves like turning the picture or shifting parts of the picture and so on which will make it look the same as the original picture?

Here is a famous tiling pattern made by the Dutch artist and mathematician Maurits C. Escher.



The famous British mathematician Roger Penrose has played with tiling. If you want to know more about this work on tiling you could look up

‘Tiling and Patterns’ by B. Grunbaum and G. C. Shephard W. H. Freeman, San Francisco, Chapter 10, 1987. One of his famous tiling patterns ‘darts and kites’ has been reproduced here.



Tiling and Stacking

There is a stunning example of tiling in nature. Many of us have seen honeycomb patterns on beehives. I have taken a paint impression from a real beehive. This is a perfect example of a hexagonal tiling pattern.

You can modify the same pattern and you have an interesting pattern of stacked earthen pots (see the picture below)

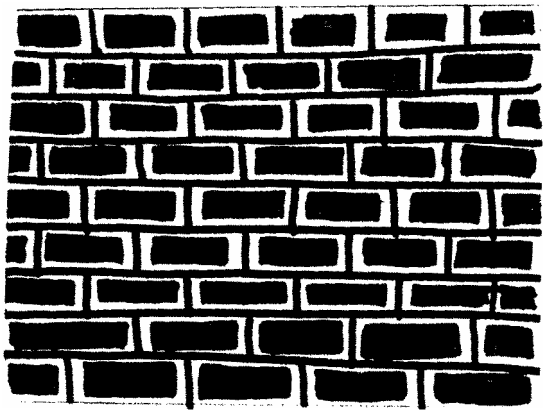
Can you figure out how I have gone from the original tiling of hexagons to the stack of pots? Check that the new basic tile can be overlaid on to the old basic hexagonal tile by rearranging the pieces. Try making your own patterns and using them to make decorated gift wrappers out of ordinary brown paper sheets.

If you are lucky you can get a piece out of a real beehive and make block printed wrappers by dipping this piece in poster colour paint. I have shown you such a paint impression of a honey comb.

In fact, in our everyday lives we see several examples of stacking. In our three dimensional world, we see roadside patterns keeping their earthen pots stacked.

The idea behind stacking or packing is similar to that behind tiling. We put things together leaving as few gaps in between. We see fruit sellers using this idea in their attractive, neat stacks of fruits at the roadside.

At building construction sites we often see scattered piles of bricks. When the bricks have been laid to build a wall, they look orderly. If you take a closer look you find that there are lots of gaps between the bricks in a pile.

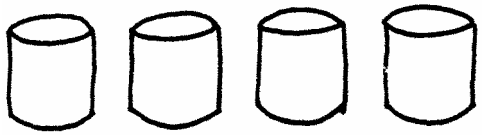


In contrast in an orderly stacking of bricks in a wall, there are hardly any gaps left between the bricks.

Here is an activity that you can try out. You need a few empty matchboxes 

and an empty cardboard box of the kind they sell tea leaves in. 

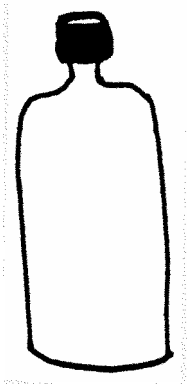
Fill up the cardboard box with these empty matchboxes. First try stacking the matchboxes in the cardboard box without bothering to arrange them neatly. Make a note of the number of matchboxes you used to fill up the cardboard box. Now take them out and fill up the box in an orderly manner with the empty matchboxes. This orderly stacking will resemble the pattern of bricks in a building wall. Count the number of matchboxes. Do you need to use more matchboxes to fill up the cardboard box when you stack them up in an orderly way? If so, how many more? This number will give you a measure of how much gap was left between the matchboxes when you tried a random stacking.

You can repeat this activity by using playing marbles 
instead of matchboxes. Here we will demonstrate a different way of measuring the amount of gap left in a random packing. Take four cylindrical glasses of the same size 

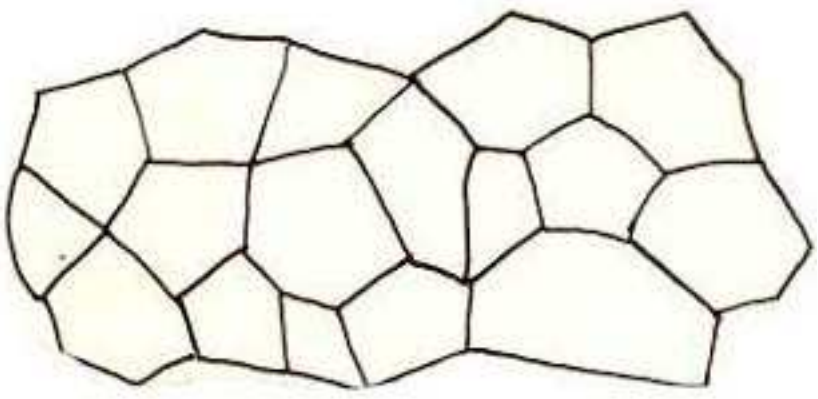
Take a few playing marbles, of the same size. Fill up a glass with these marbles, without bothering to stack them neatly. Pour water into this glass to measure the total amount of space left between the marbles. Pour the water into an empty glass. Now take another glass and fill it up with marbles in an orderly manner. Fill this glass up with water. Now compare the water levels in these two cases by emptying the water out into the fourth glass. The difference in the water level in the two glasses will give you a measure of the extra space left between the marbles in a random packing.

Do you know that many substances in nature are made up of organised structures of repeated building bricks? Such organised structures are called crystals. Common examples of crystals are common salt and sugar. A less common (and more expensive!) example is diamond. Each grain of salt is made up of many identical cubic 'bricks' arranged regularly.

Here is another activity that you can try out. Take a glass or a transparent plastic bottle of shampoo that is nearly empty. Just with a drop of shampoo in this bottle you can form interesting foam structures by adding water and shaking the bottle after putting the cap on. The patterns you see in the shampoo foam typically look like the picture shown below. This is an example of a three dimensional packed structure. Each unit in this foam structure looks like a many fold three dimensional object (an example of such an object is for instance, a cube). You will notice that there are many gaps left between two adjacent three dimensional foam structures. In this particular case, the foam structures filling up the bottle are not identical.



In this picture of such a foam structure, the basic building units look two dimensional, but they are really three dimensional. You can just do it and see yourself.



The main difference between a regular crystalline structure and such a foam structure is that here, unlike a crystal, a basic unit is not repeated. But we do get a structure with no gaps between the building blocks and because the bottle and the foam are transparent we can actually see a three dimensional packed structure with very little effort!

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